

# Segway and the Graphical Models Toolkit: A framework for probabilistic genomic inference

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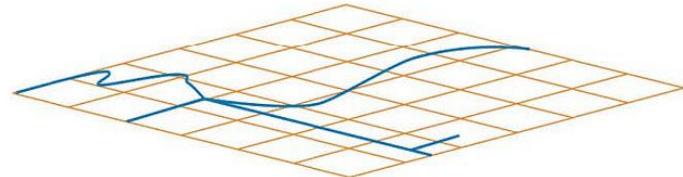
@michaelhoffman

# Geographical maps have...

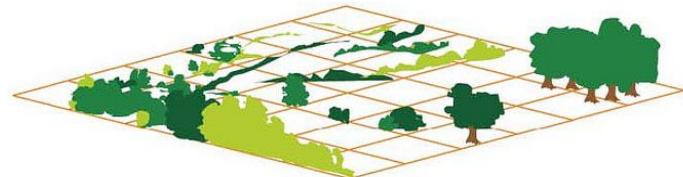
street data



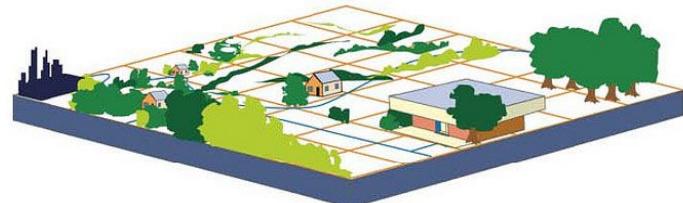
+ buildings data



+ vegetation data



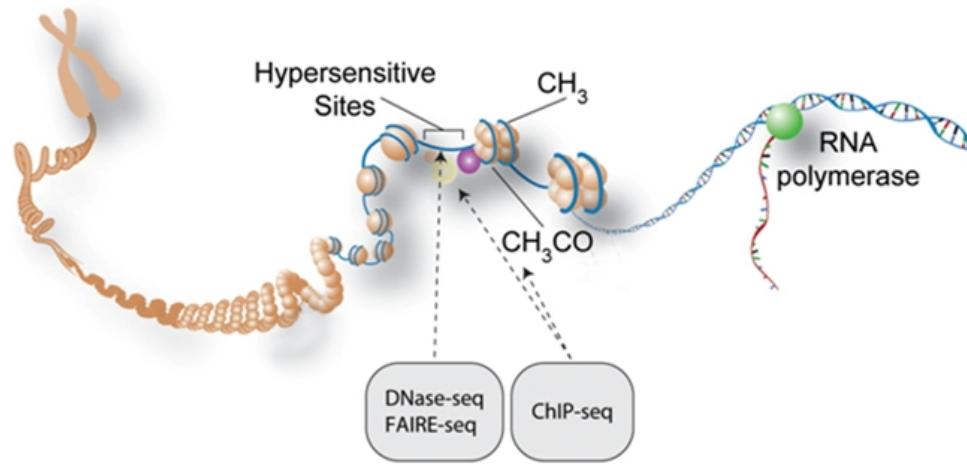
= integrated map



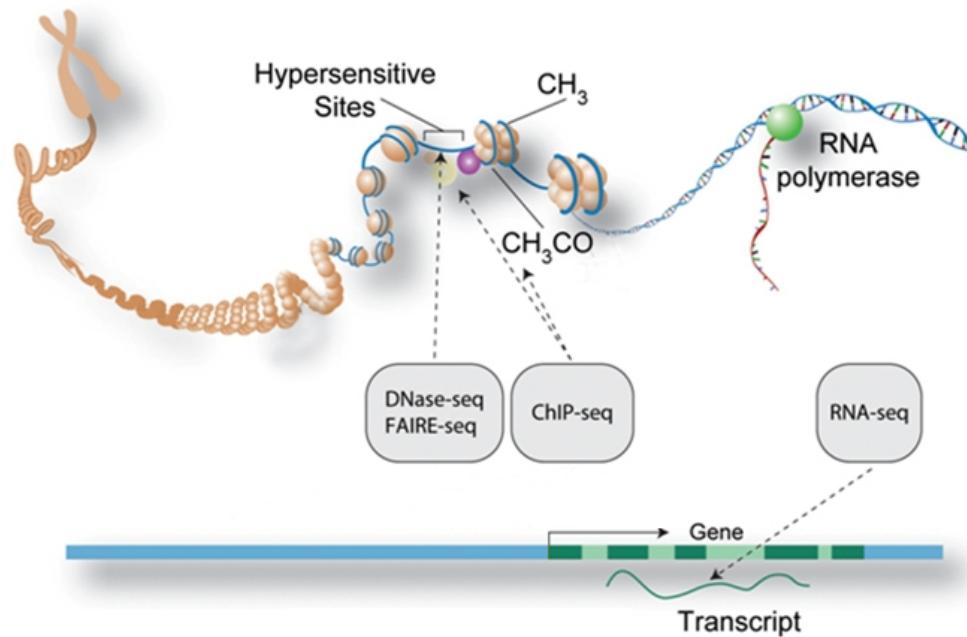
# Functional genomics



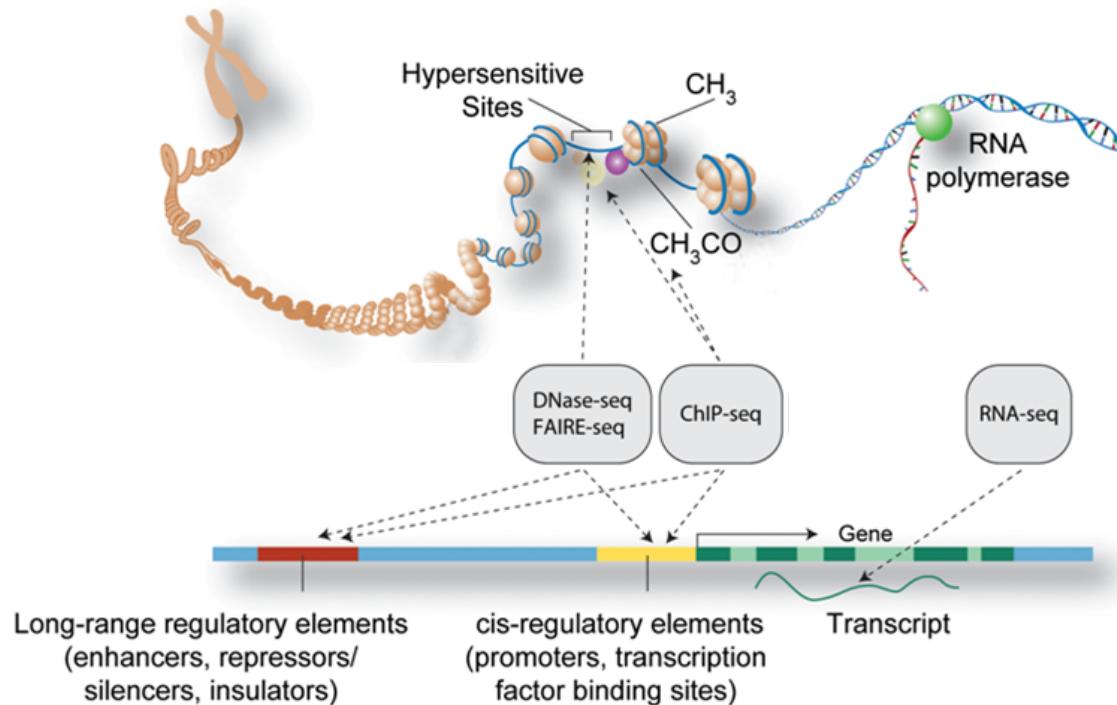
# Functional genomics



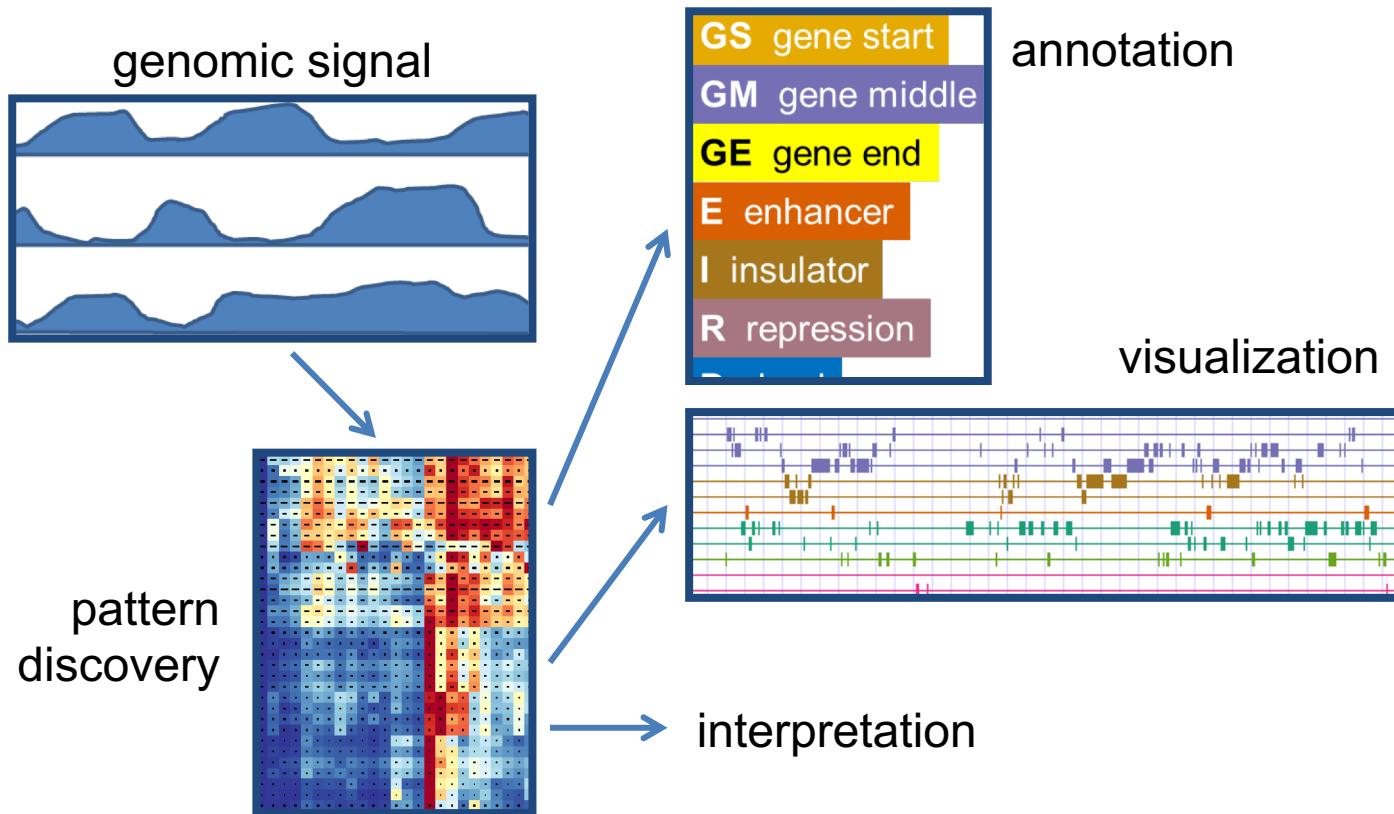
# Functional genomics



# Functional genomics



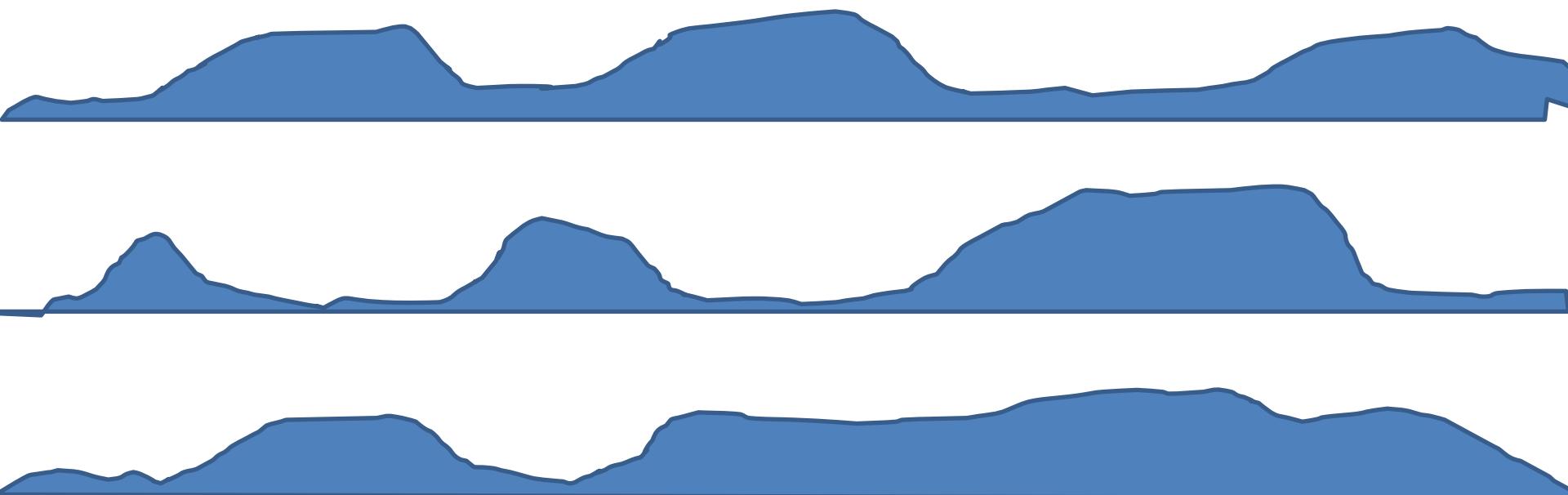
# Semi-automated genome annotation



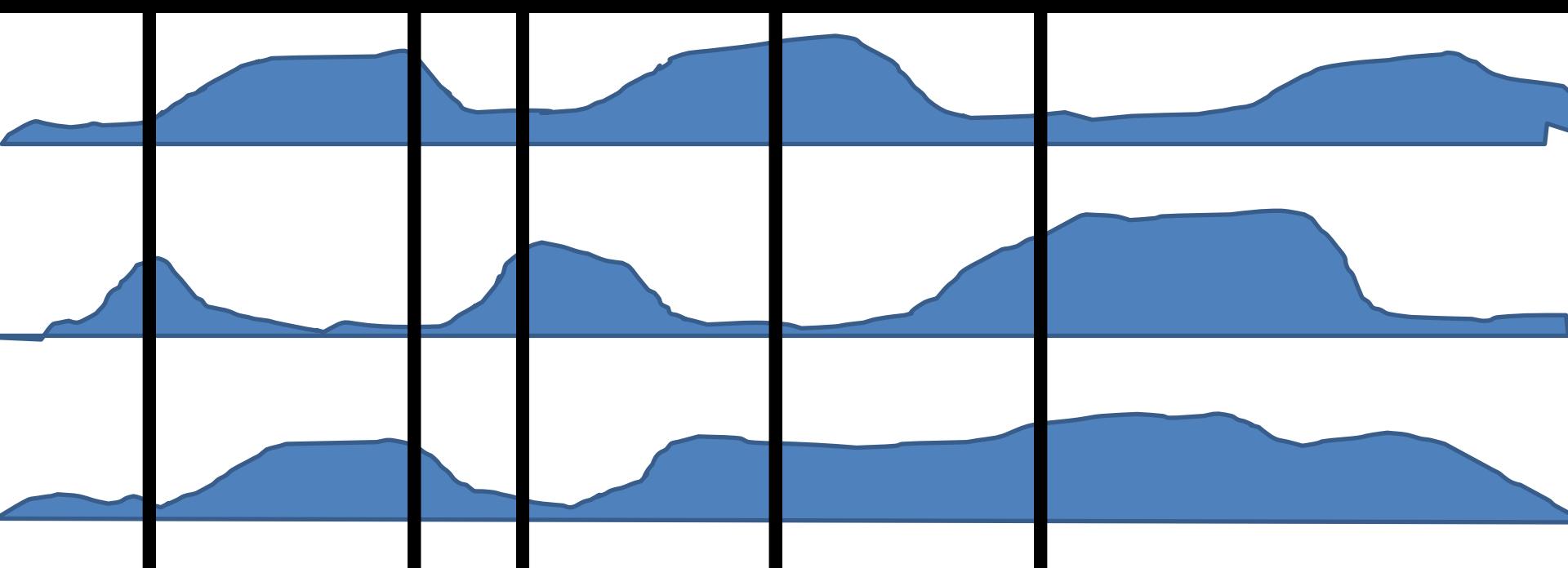
# Segway

A way to segment the genome

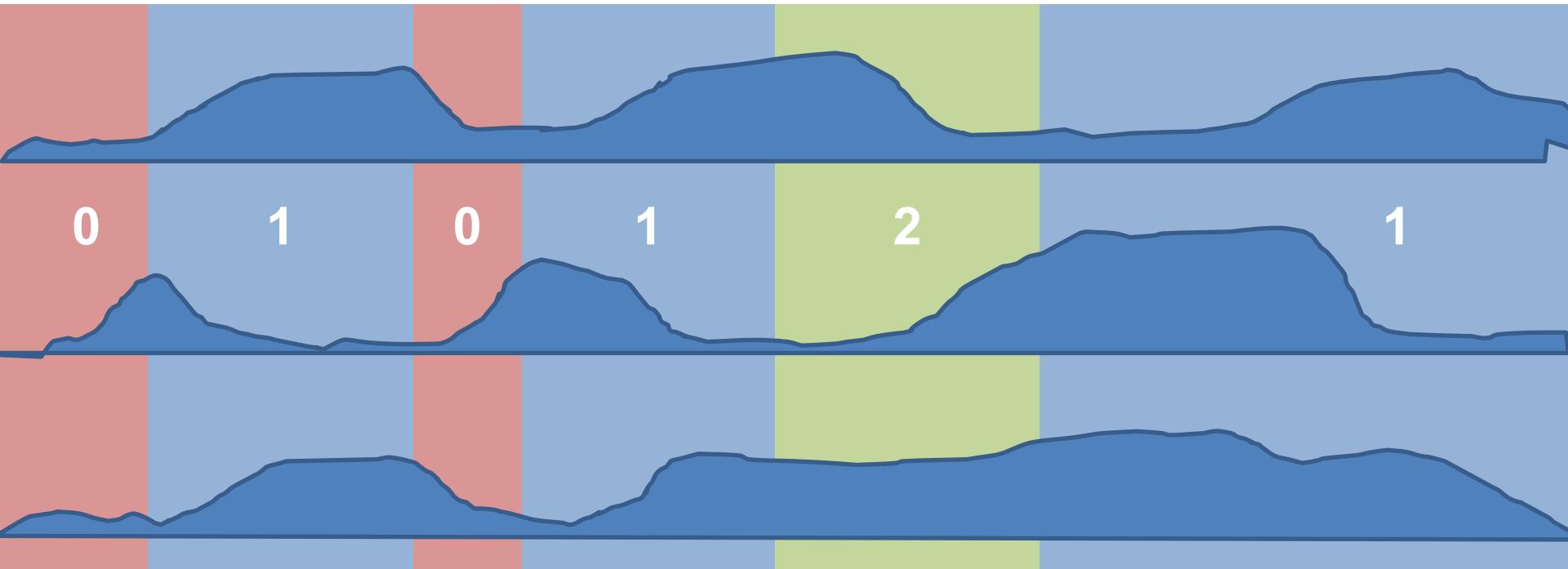
# Genomic segmentation



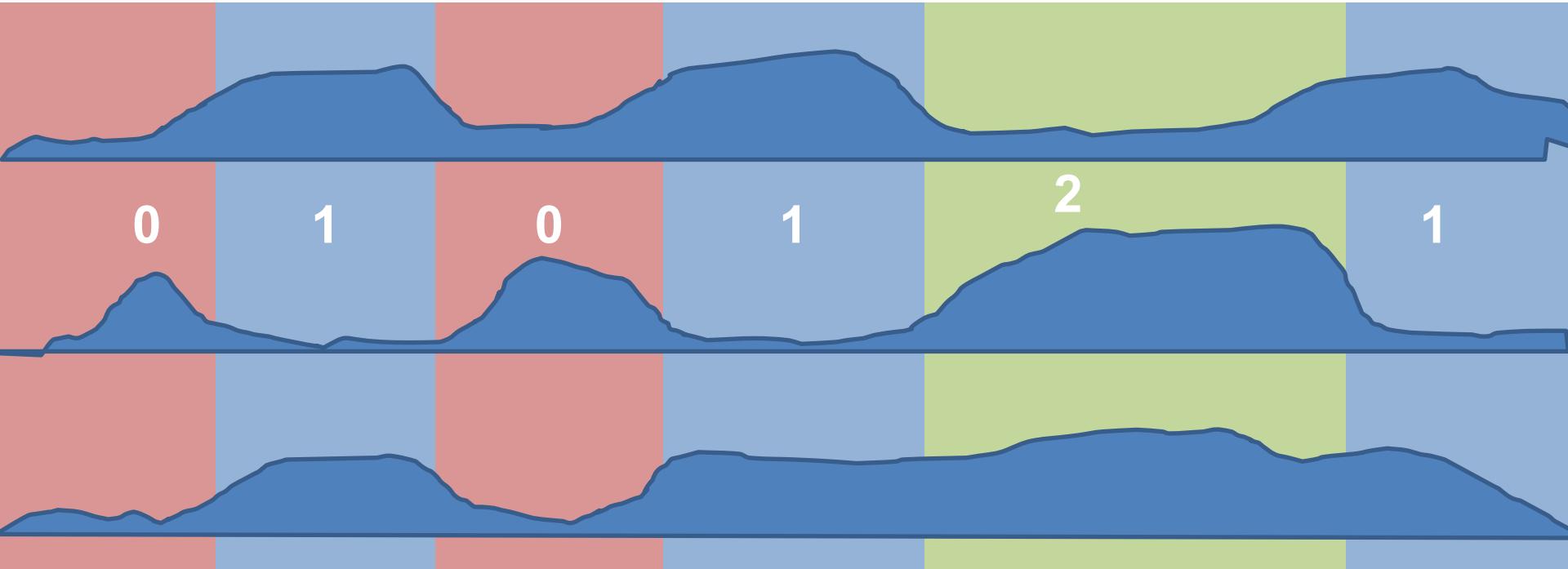
# Nonoverlapping segments



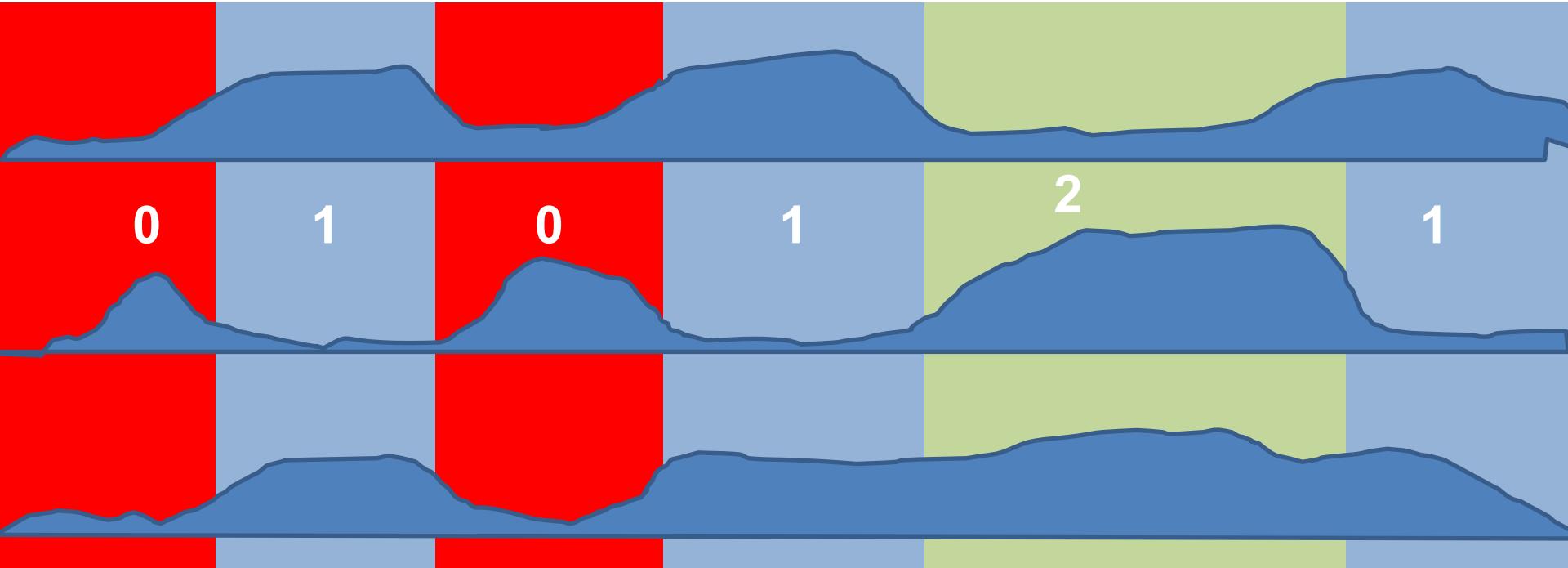
# Finite number of labels



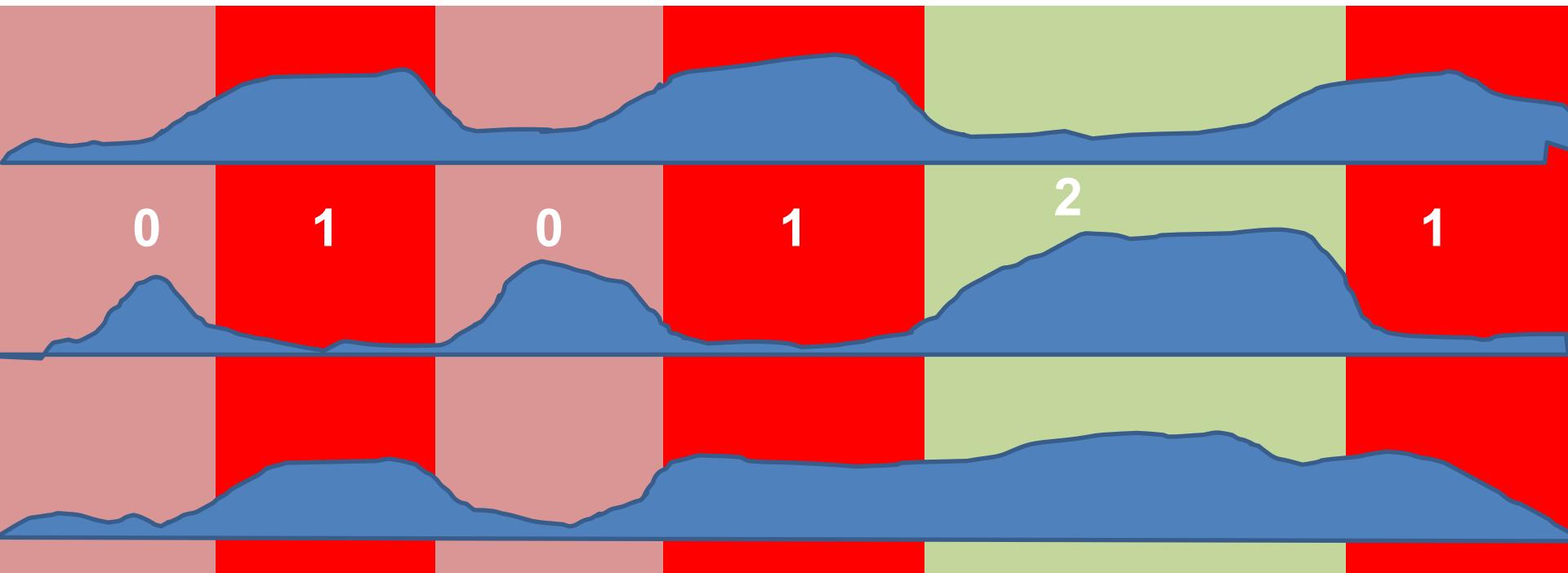
# Maximize similarity in labels

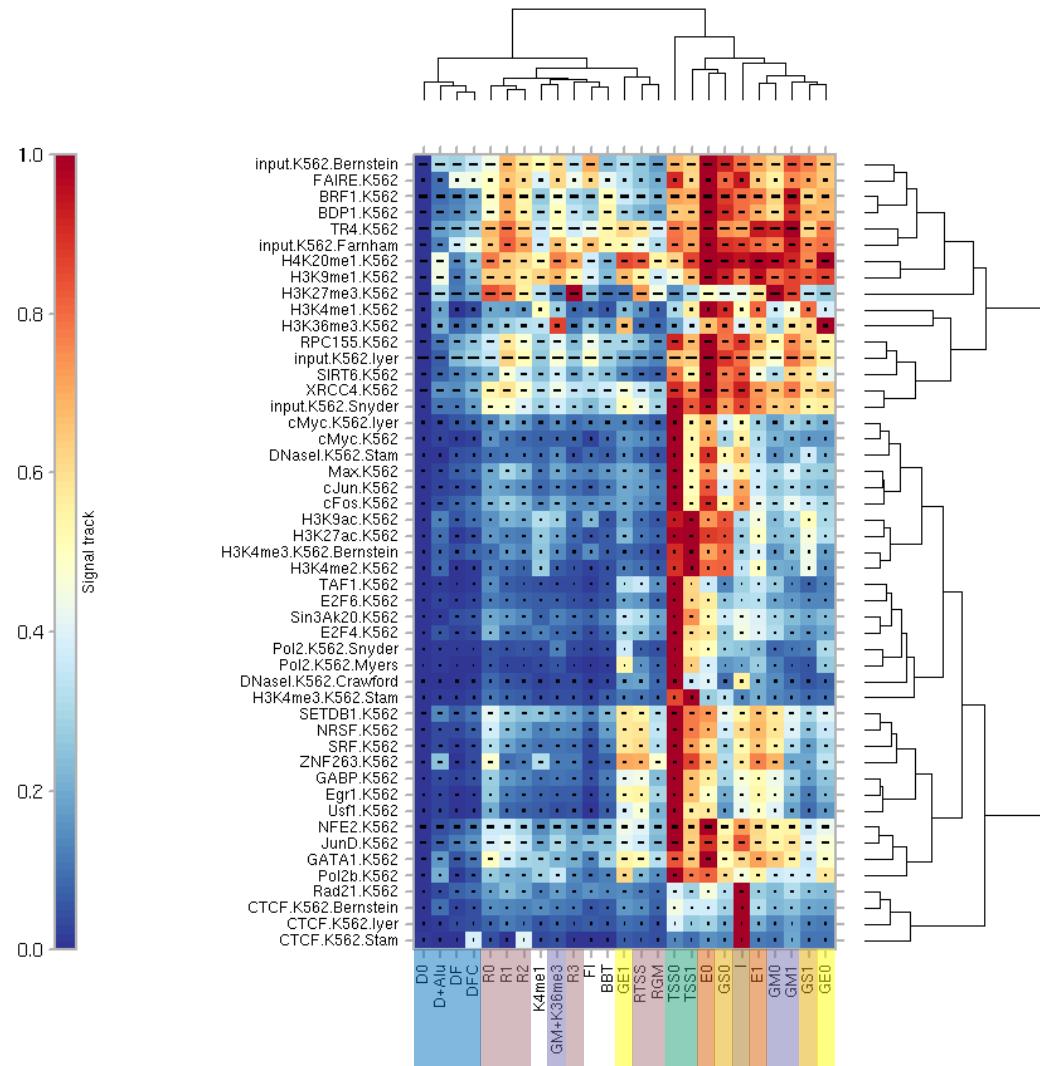


# Maximize similarity in labels



# Maximize similarity in labels





**TSS** transcription start site

**GS** gene start

**GM** gene middle

**GE** gene end

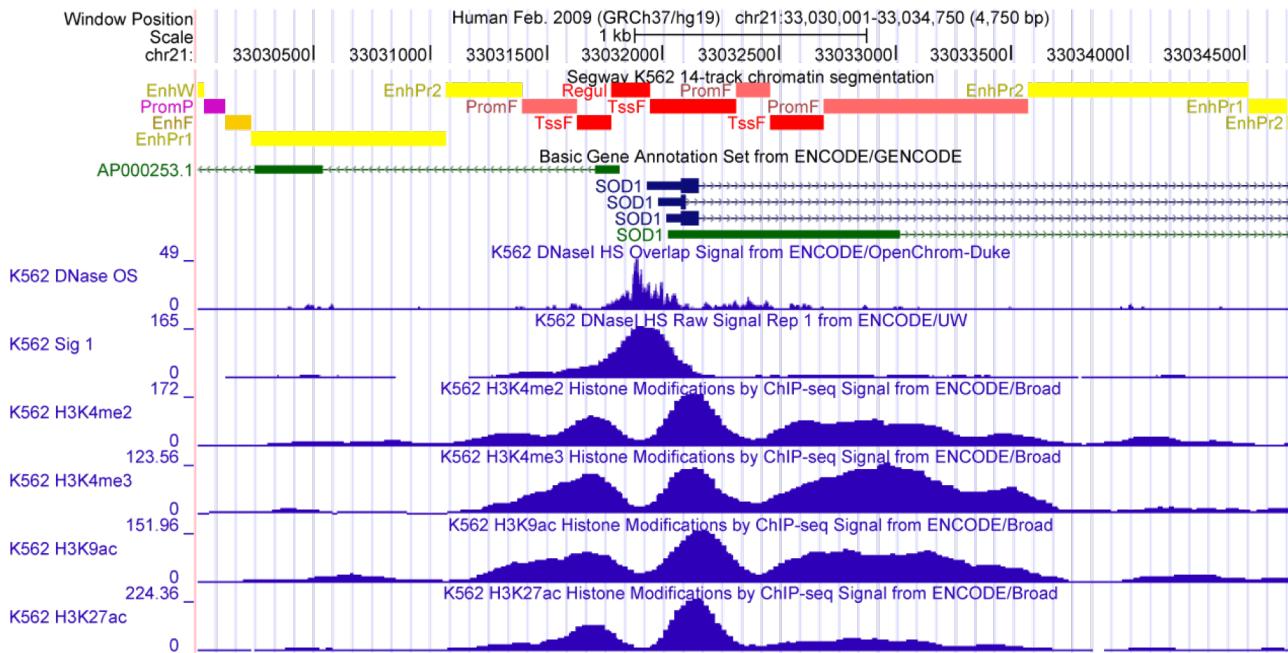
**E** enhancer

**I** distal CTCF

**R** repression

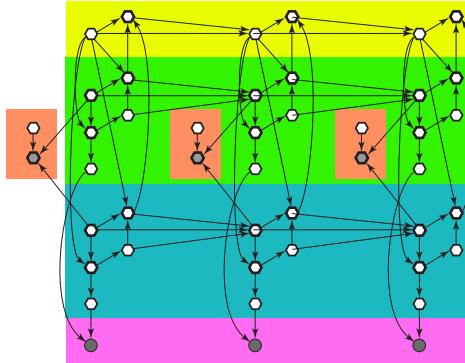
**D** dead

# Transcription start site (TSS)



# Graphical Models Toolkit (GMTK)

- General purpose toolkit for probabilistic inference in temporal signals (speech, language, activity recognition, genomics)
- Written in highly optimized scalable C++
- Supports many machine learning algorithms, inference procedures, and probability models.
- Can express arbitrary structured dynamic graphical models
- <http://melodi.ee.washington.edu/gmtk>
- `conda install -c bioconda gmtk`



Jeff Bilmes

Q

# Graphical



hidden random variable



discrete

Q

# Graphical



hidden random variable



discrete

realization      parameters

$$P(q | \theta) = P(Q = q)$$

# Equation

```
variable: state {  
    type: discrete hidden cardinality 2;  
    conditionalparents: nil using DenseCPT("start_state");  
}
```

GMTKL

# Graphical



hidden random variable



discrete

$$P(q | \theta) = P(Q = q)$$

Equation

Q

# Graphical



hidden random variable



discrete

```
variable: state {  
    type: discrete hidden cardinality 2;  
    conditionalparents: nil using DenseCPT("start_state");  
}
```

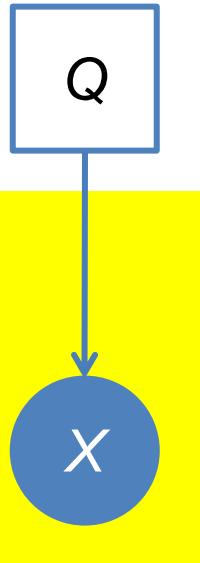
# GMTKL

```
DENSE_CPT_IN_FILE inline % conditional probability tables  
1 % total number of CPTs = 1  
  
0 start_state % CPT #0, "start_state"  
0 % 0 parents  
2 % output cardinality 2  
0.25 % P(Q = 0) = 0.25  
0.75 % P(Q = 1) = 0.75
```

# Parameters

$$P(q | \theta) = P(Q = q)$$

# Equation



```

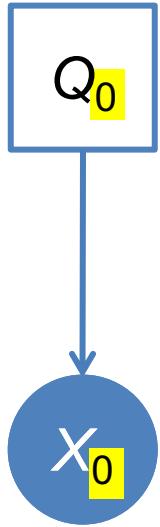
variable: state {
    type: discrete hidden cardinality 2;
    conditionalparents: nil using DenseCPT("start_state");
}

variable: obs {
    type: continuous observed 0:0;
    conditionalparents: state(0) using mapping("state_obs");
}

```

- hidden random variable
- observed random variable
- discrete    continuous
- conditional relationship

$$\begin{aligned}
 P(q, x | \theta) \\
 = P(Q = q)P(X = x | Q = q)
 \end{aligned}$$

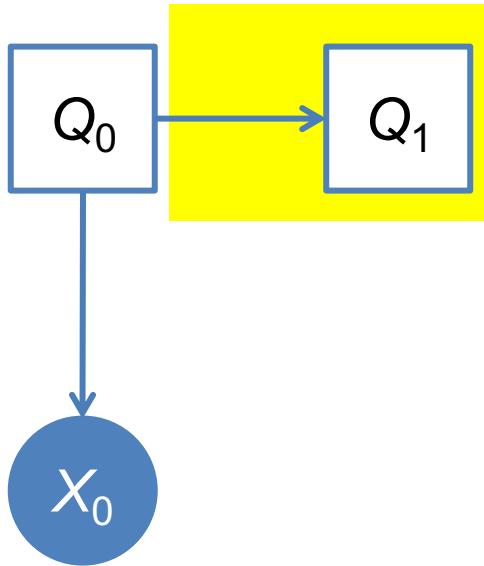


- hidden random variable
- observed random variable
- discrete
- continuous
- conditional relationship

```

frame: 0 {
    variable: state {
        type: discrete hidden cardinality 2;
        conditionalparents: nil using DenseCPT("start_state");
    }
    variable: obs {
        type: continuous observed 0:0;
        conditionalparents: state(0) using mapping("state_obs");
    }
}
  
```

$$\begin{aligned}
 P(q_0, x_0 | \theta) \\
 = P(Q_0 = q_0)P(X_0 = x_0 | Q_0 = q_0)
 \end{aligned}$$



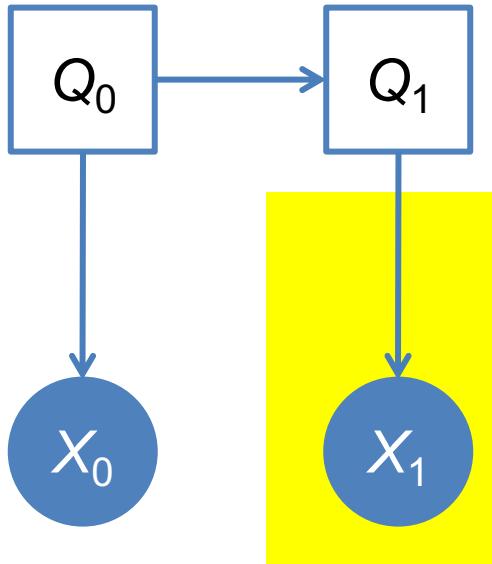
```

frame: 0 {
    variable: state {
        type: discrete hidden cardinality 2;
        conditionalparents: nil using DenseCPT("start_state");
    }
    variable: obs {
        type: continuous observed 0:0;
        conditionalparents: state(0) using mapping("state_obs");
    }
}
frame: 1 {
    variable: state {
        type: discrete hidden cardinality 2;
        conditionalparents: state(-1) using DenseCPT("state_state");
    }
}

```

- hidden random variable
- observed random variable
- discrete
- continuous
- conditional relationship

$$\begin{aligned}
 P(q_{0:1}, x_0 | \theta) \\
 = P(Q_0 = q_0)P(X_0 = x_0 | Q_0 = q_0)P(Q_1 = q_1 | Q_0 = q_0)
 \end{aligned}$$



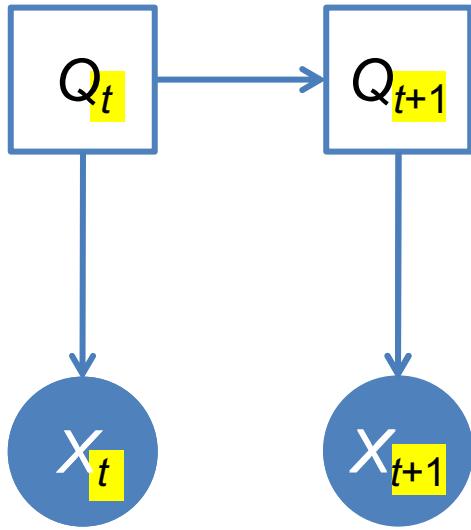
- hidden random variable
- observed random variable
- discrete   ● continuous
- ↓ conditional relationship

```

frame: 0 {
    variable: state {
        type: discrete hidden cardinality 2;
        conditionalparents: nil using DenseCPT("start_state");
    }
    variable: obs {
        type: continuous observed 0:0;
        conditionalparents: state(0) using mapping("state_obs");
    }
}
frame: 1 {
    variable: state {
        type: discrete hidden cardinality 2;
        conditionalparents: state(-1) using DenseCPT("state_state");
    }
    variable: obs {
        type: continuous observed 0:0;
        conditionalparents: state(0) using mapping("state_obs");
    }
}

```

$$\begin{aligned}
& P(q_{0:1}, x_{0:1} | \theta) \\
& = P(Q_0 = q_0)P(X_0 = x_0 | Q_0 = q_0)P(Q_1 = q_1 | Q_0 = q_0)P(X_1 = x_1 | Q_1 = q_1)
\end{aligned}$$



- hidden random variable
- observed random variable
- discrete
- continuous
- ↓ conditional relationship

```

frame: 0 {
    variable: state {
        type: discrete hidden cardinality 2;
        conditionalparents: nil using DenseCPT("start_state");
    }
    variable: obs {
        type: continuous observed 0:0;
        conditionalparents: state(0) using mapping("state_obs");
    }
}
frame: 1 {
    variable: state {
        type: discrete hidden cardinality 2;
        conditionalparents: state(-1) using DenseCPT("state_state");
    }
    variable: obs {
        type: continuous observed 0:0;
        conditionalparents: state(0) using mapping("state_obs");
    }
}

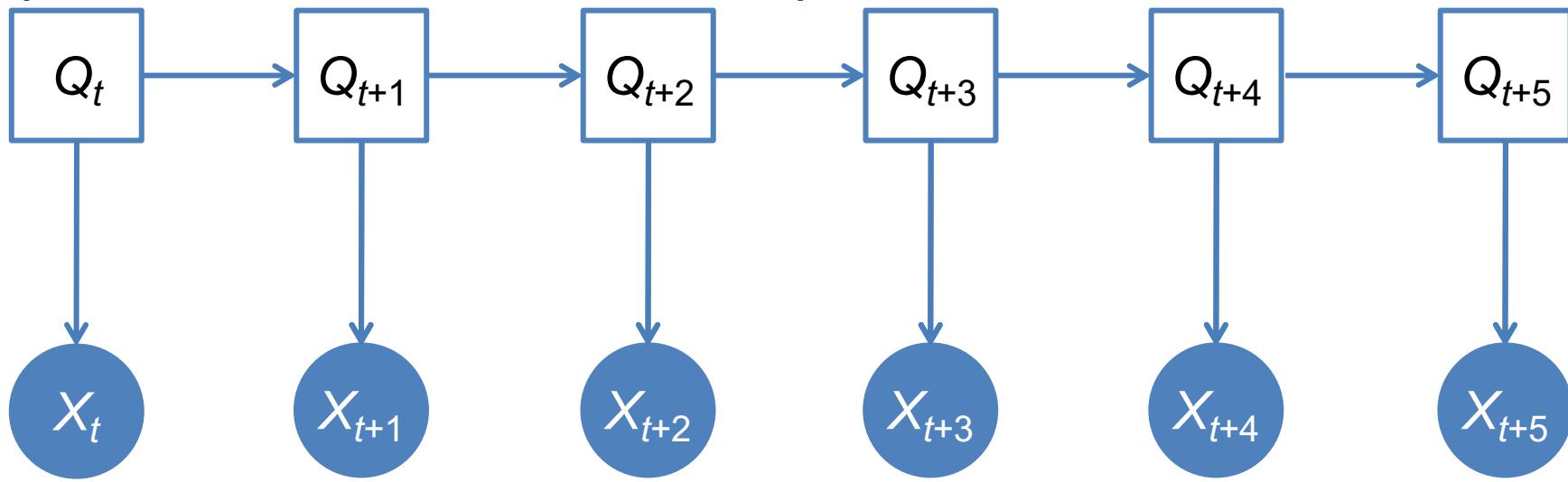
```

chunk 1:1

$$P(q_{0:T}, x_{0:T} | \theta)$$

$$= P(Q_0 = q_0) P(X_0 = x_0 | Q_0 = q_0) \prod_{t=1}^T P(Q_t = q_t | Q_{t-1} = q_{t-1}) P(X_t = x_t | Q_t = q_t)$$

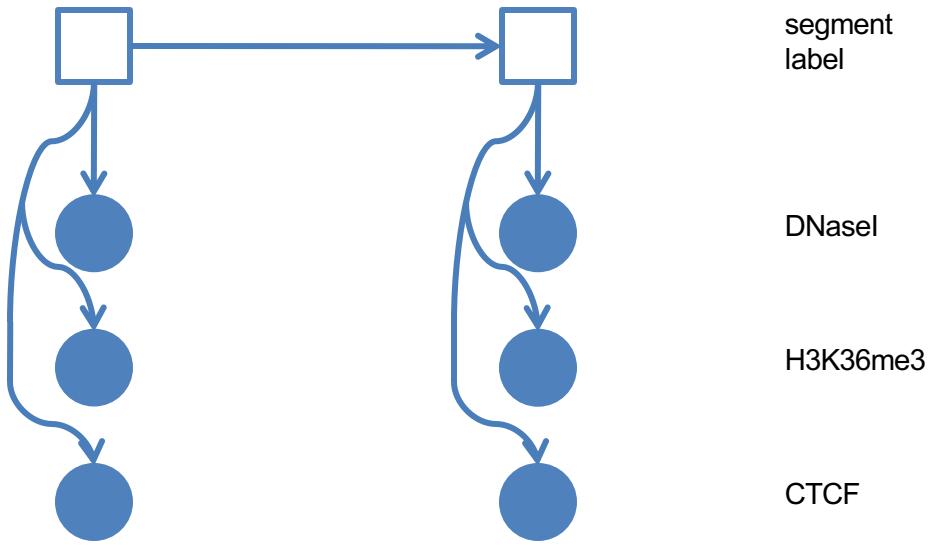
# Dynamic Bayesian network (hidden Markov model)



- hidden random variable
- observed random variable
- discrete   ● continuous
- ↙ conditional relationship

$$\begin{aligned} P(q_{0:T}, x_{0:T} | \theta) \\ = P(Q_0 = q_0)P(X_0 = x_0 | Q_0 = q_0) \prod_{t=1}^T P(Q_t = q_t | Q_{t-1} = q_{t-1})P(X_t = x_t | Q_t = q_t) \end{aligned}$$

# Dynamic Bayesian Network for segmentation



hidden random variable



observed random variable



discrete

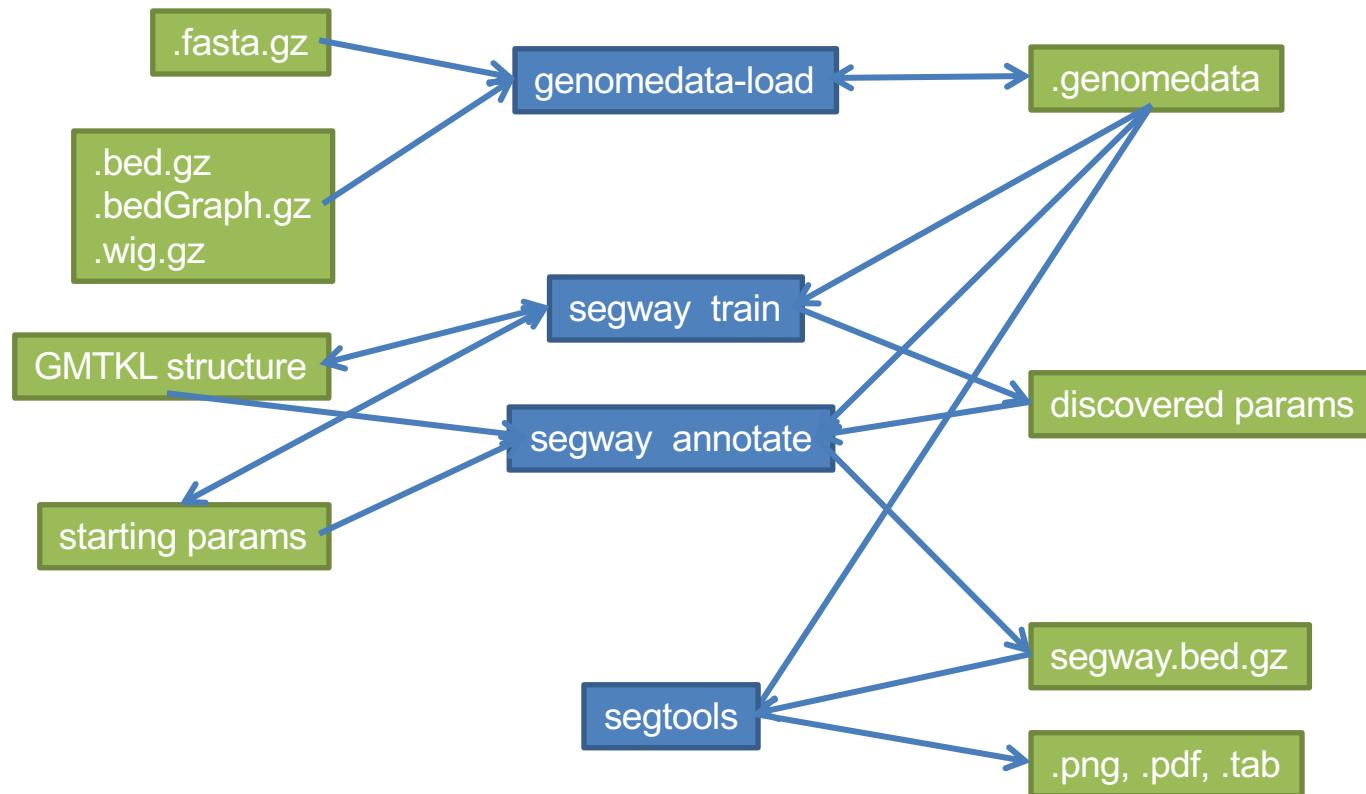


continuous



conditional relationship

# Segway work flow



# What does Segway do, really?

1. Creates GMTKL structure and parameter files for semi-automated genome annotation
2. Converts genomic data to a GMTK binary observation format
3. Runs GMTK to perform EM training, Viterbi decoding, posterior inference
4. Manages job execution via cluster (SGE, LSF, PBS, Slurm), or local multiprocessing
5. Converts GMTK output to genomics formats (BED, wiggle)

# Segway modular API (pull request #91)

segway train

segway train-init

segway train-run

segway train-run-round

segway train-finish

segway annotate

segway annotate-init

segway annotate-run

segway annotate-finish

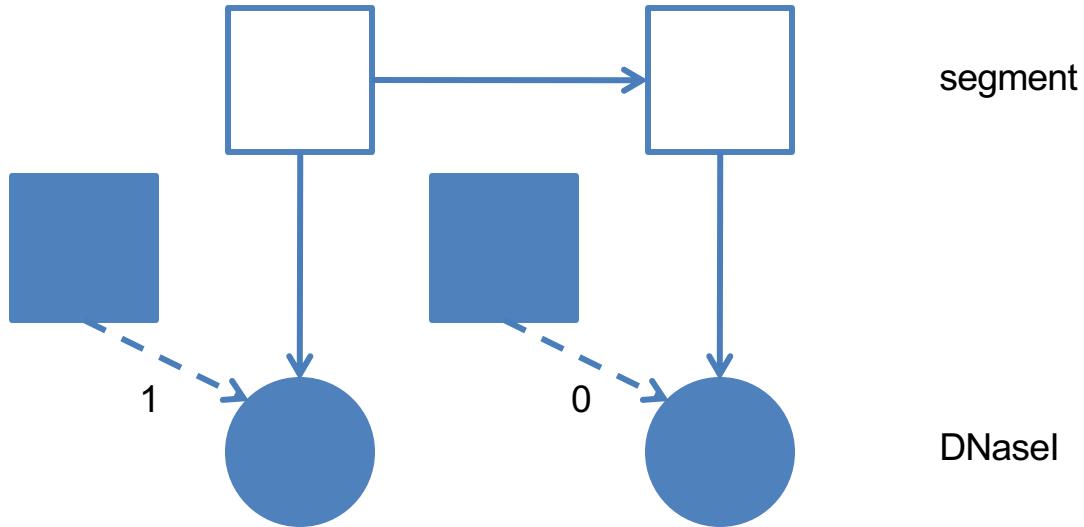
segway posterior

segway posterior-init

segway posterior-run

segway posterior-finish

# Handling missing data



hidden random variable



observed random variable



discrete



continuous

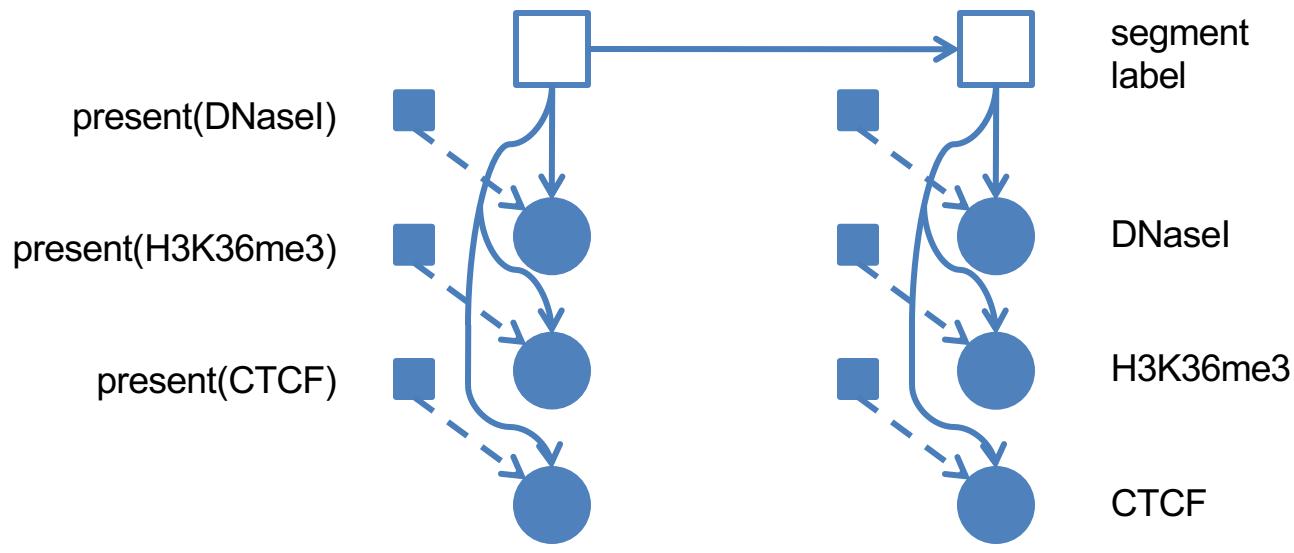


conditional



switching

# Handling missing data



hidden random variable



observed random variable



discrete



continuous

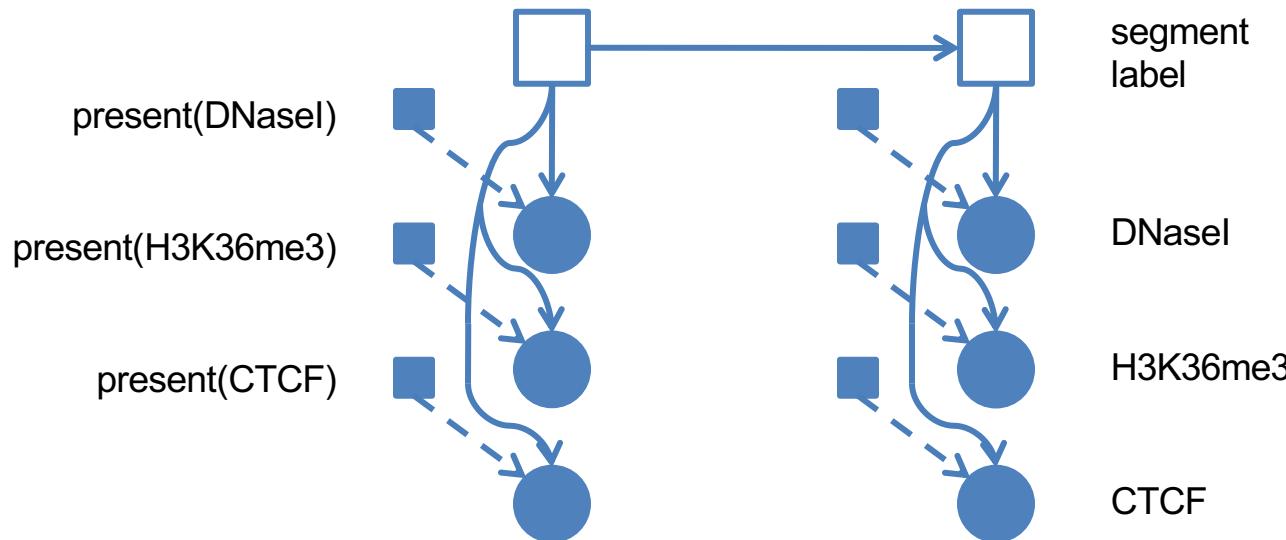


conditional

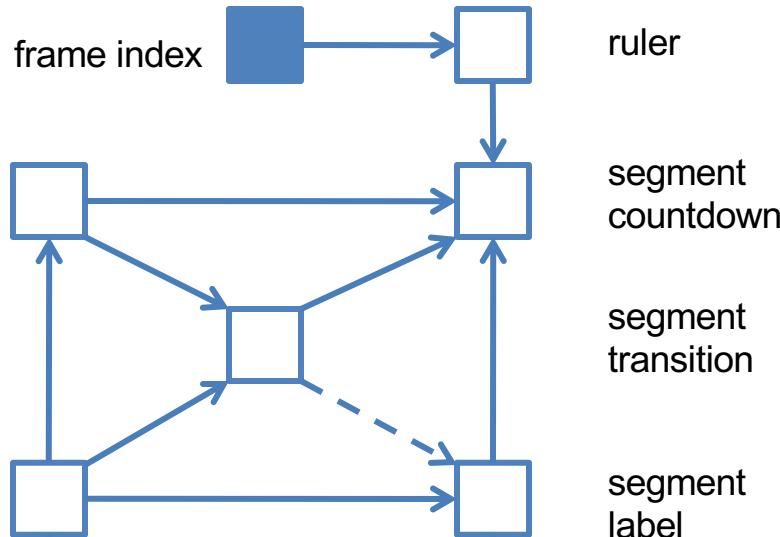


switching

# Length distribution

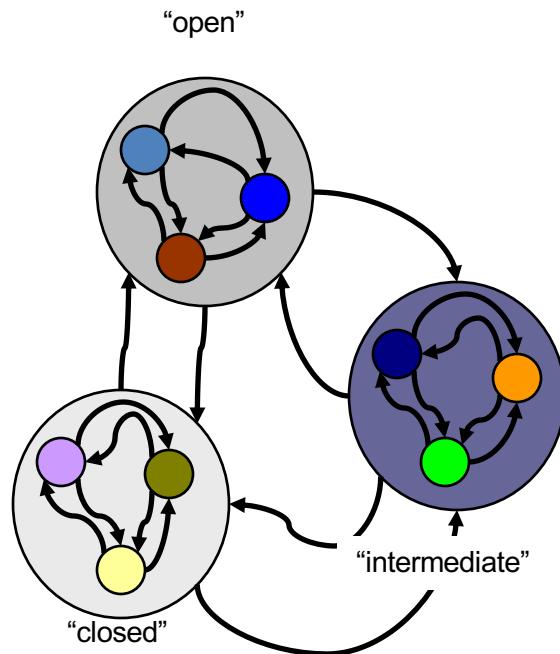


# Length distribution

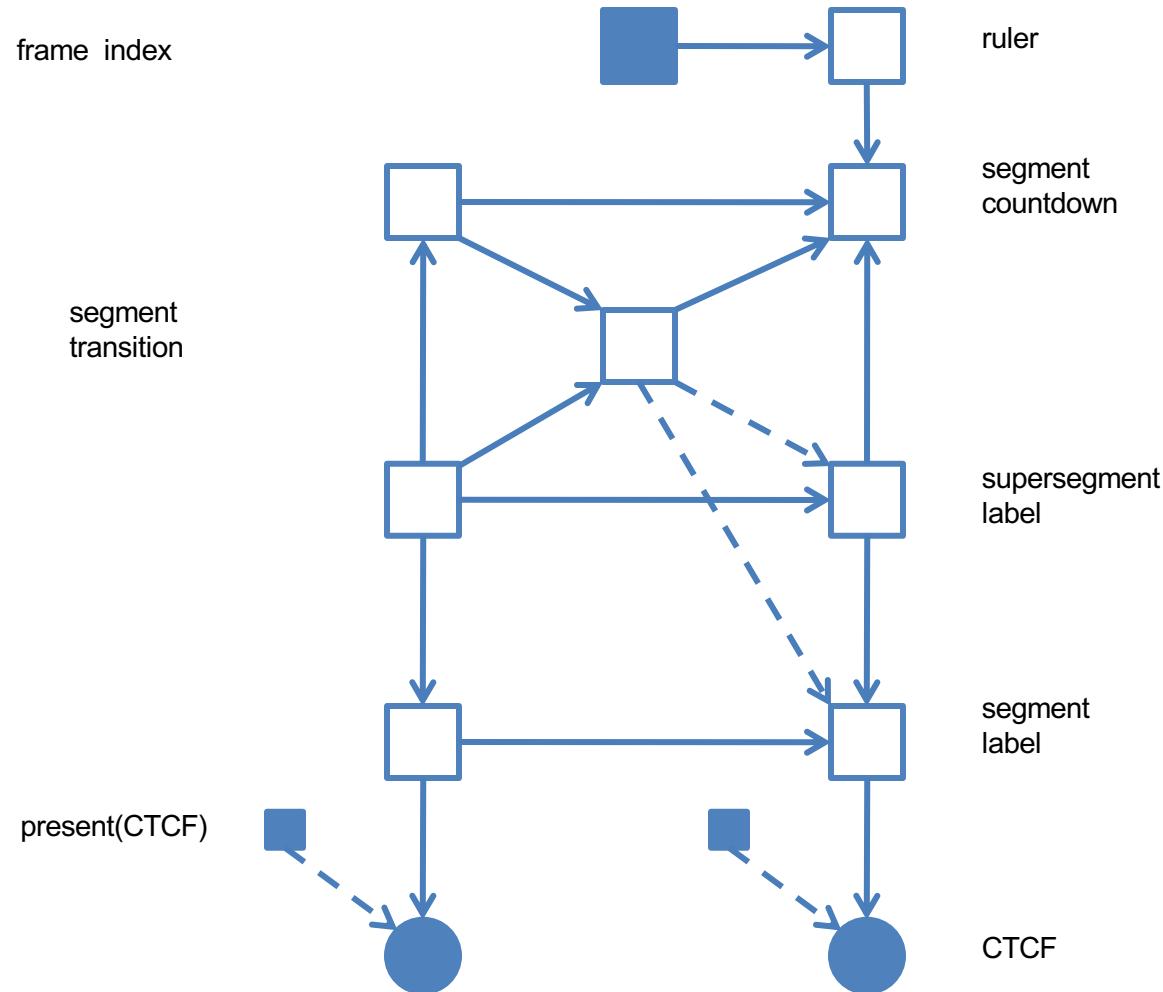


- Minimum segment length
  - Maximum segment length
  - Trained geometric length distribution
  - Dirichlet prior on segment length
  - Weight of prior versus observed data
- present(DNasel)      present(H3K36me3)      present(CTCF)
- DNasel      H3K36me3      CTCF

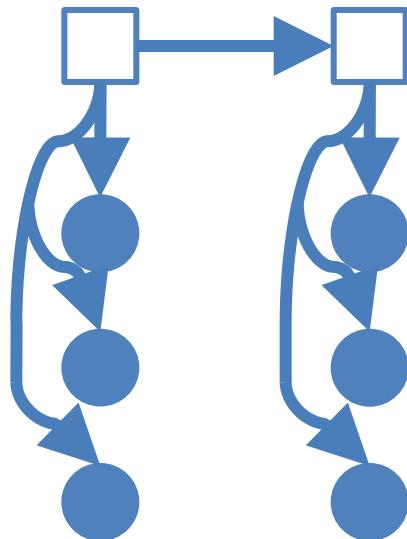
# Hierarchical models



- Two-level model topology
- Mixture of large-scale and punctate labels



+ strand



segment  
label

CAGE  
nucleus

short RNA-seq  
whole cell

long RNA-seq  
cytosol



hidden random variable



observed random variable



discrete

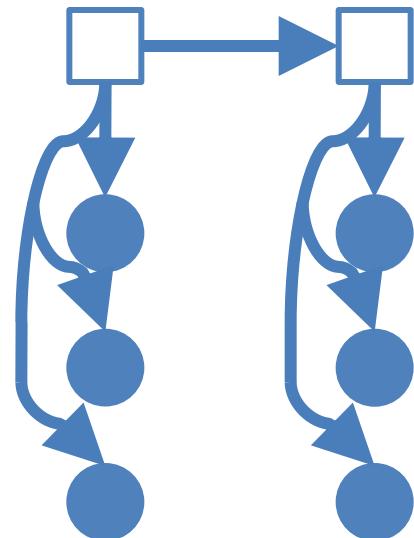


continuous

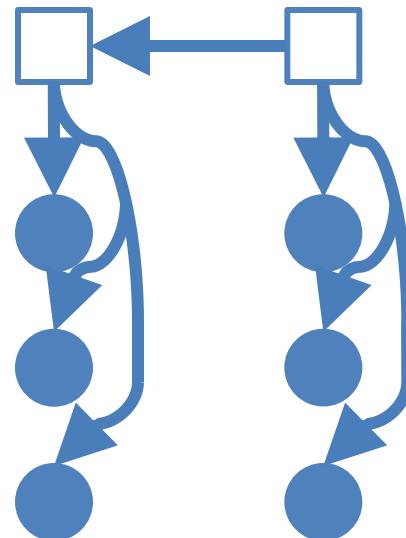


conditional relationship

+ strand



- strand



segment  
label

CAGE  
nucleus

short RNA-seq  
whole cell

long RNA-seq  
cytosol



hidden random variable



observed random variable



discrete

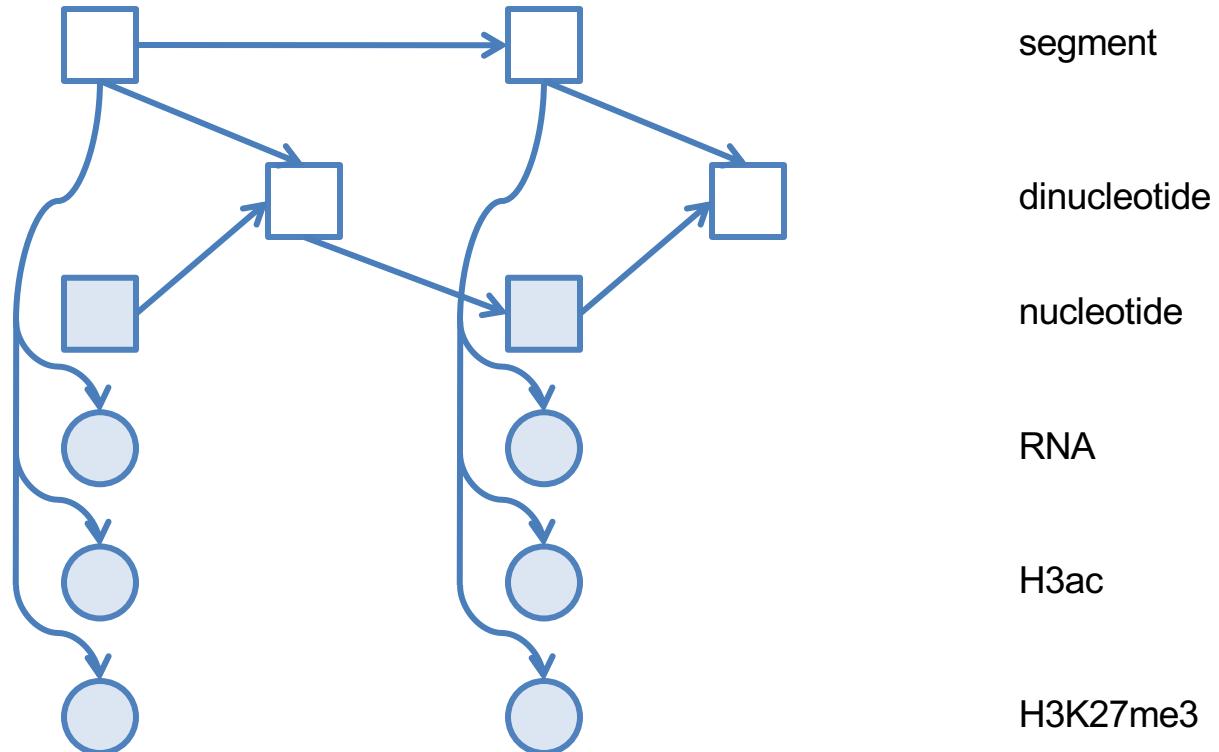


continuous

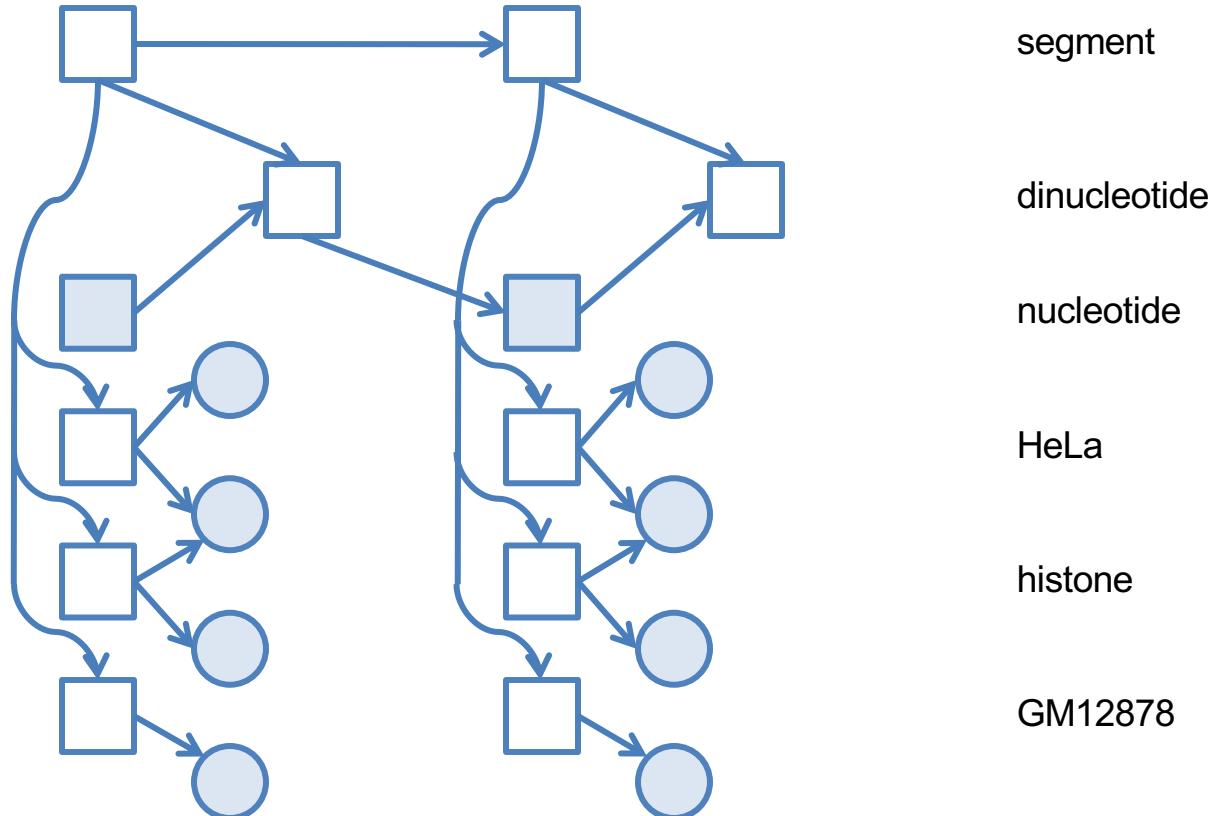


conditional relationship

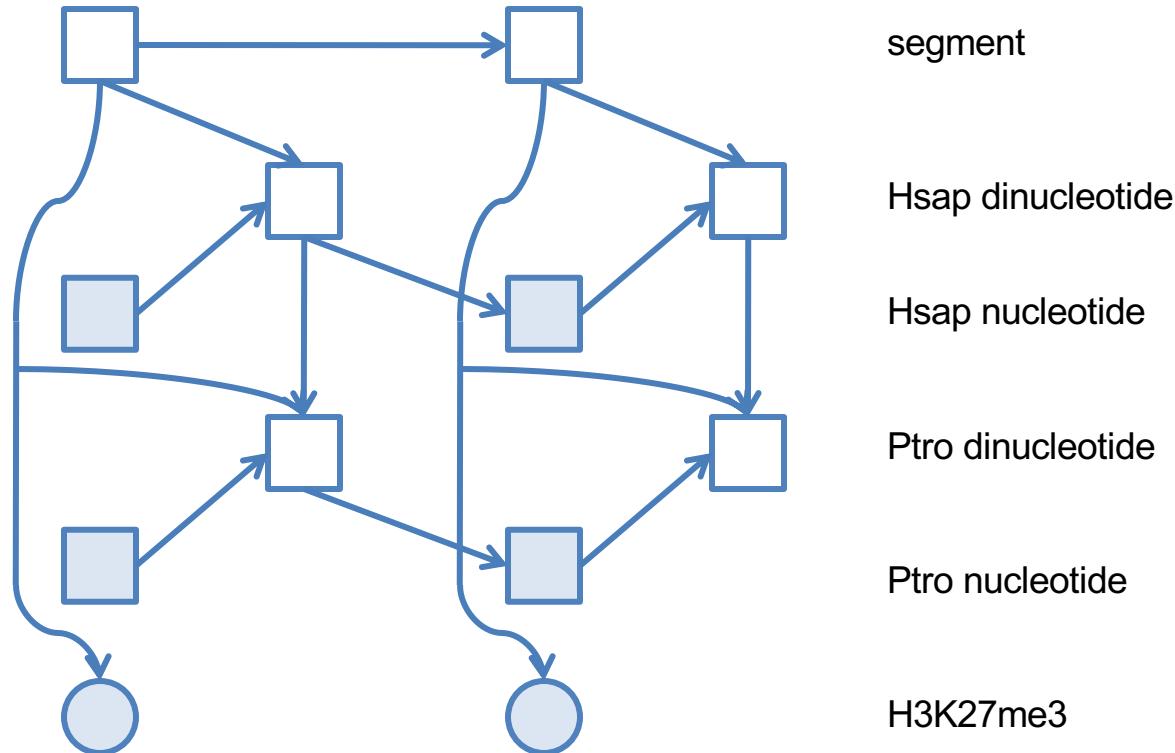
# Adding genome sequence



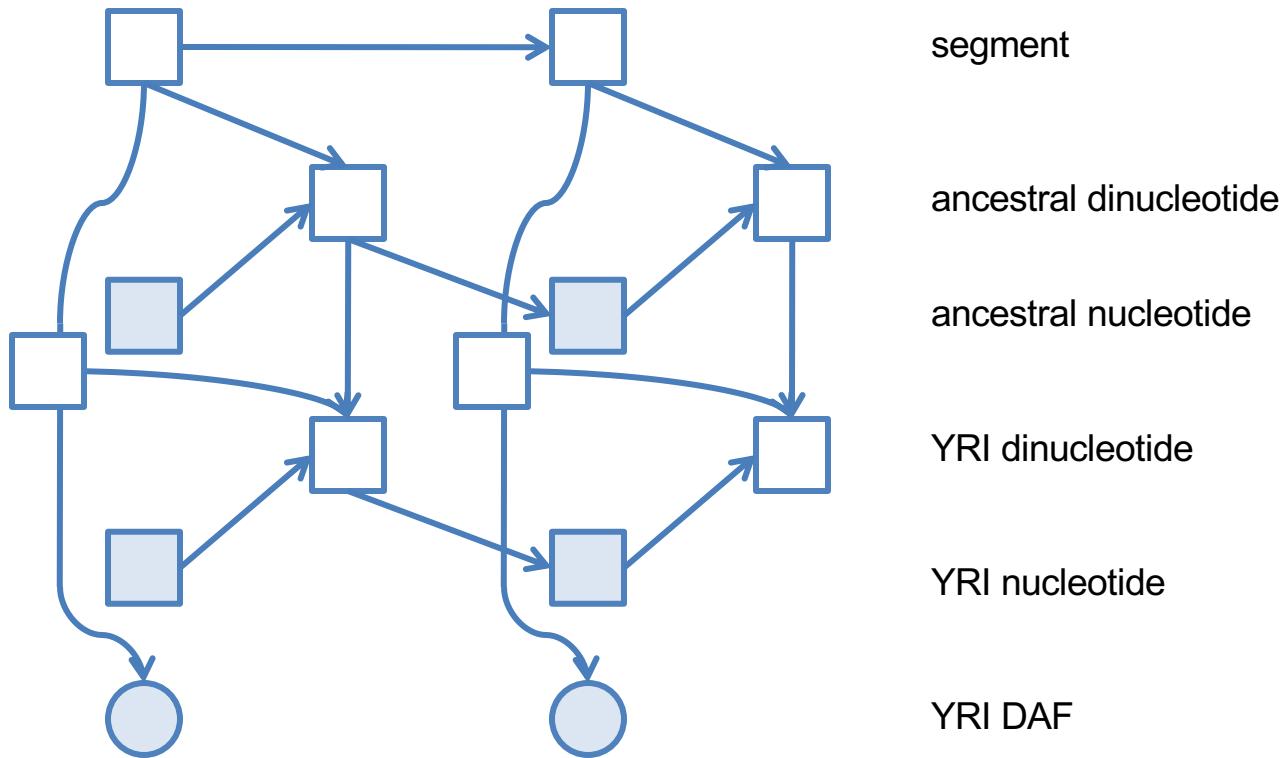
# Adding data relationships



# Adding evolution



# Adding variation



Segway: semi-automated genomic annotation – Mozilla Firefox

Segway: semi-automate... +

https://www.pmgenomics.ca/hoffmanlab/proj/segway/ Search

# Segway semi-automated genomic annotation

Hoffman MM, Buske OJ, Wang J, Weng Z, Bilmes J, Noble WS. 2012. [Unsupervised pattern discovery in human chromatin structure through genomic segmentation](#). *Nat Methods* 9:473–476. doi:10.1038/nmeth.1937. PubMed Central (free version): PMC3340533 (BibTeX)

Hoffman MM\*, Ernst J\*, Steven WP, Kundaje A, Harris RS, Libbrecht M, Giardine B, Ellenbogen PM, Bilmes JA, Birney E, Hardison RC, Dunham I, Kellis M, Noble WS. 2012. [Integrative annotation of chromatin elements from ENCODE data](#). *Nucleic Acids Res* 41:827–841 doi: (BibTeX)

**The free Segway software package contains a novel method for analyzing multiple tracks of functional genomics data.** Our method uses a dynamic Bayesian network (DBN) model, which enables it to analyze the entire genome at 1-bp resolution even in the face of heterogeneous patterns of missing data. This method is the first application of DBN techniques to genome-scale data and the first genomic segmentation method designed for use with the maximum resolution data available from ChIP-seq experiments without downsampling. Segway uses the [Graphical Models Toolkit \(GMTK\)](#) for efficient DBN inference. Our software has extensive documentation and was designed from the outset with external users in mind.

## Segmentations

### Human chromatin structure

There are two published segmentations of human chromatin structure available.

1. The regulatory segmentation from the [Ensembl Regulatory Build](#) viewable in [Ensembl](#)
2. The segmentation from our *Nature Methods* paper, "Unsupervised pattern discovery in human chromatin structure through genomic segmentation," viewable in the [UCSC Genome Browser](#)

#### Ensembl

The segmentation can be displayed by clicking the "Configure this page" option on the left navigation bar. The segmentations for each cell line can be selected under "Regulatory Features" and under the heading of "Enable/disable all Segmentation features". As an example you can try viewing the segmentations for [BRCA2](#) in hg38.

For more details and instructions see the description of [Regulatory Segmentation](#).

#### UCSC Genome Browser

The [Ensembl Regulatory Build for GRCh38 \(hg38\)](#) can be viewed [here](#). It can also be loaded through the [Track Data Hub](#) interface. You can connect "Ensembl Regulatory Build" listed in the Public Hubs directory. After loading the track hub, you can show the "Cell Type Segmentations" supertrack which contains a Segway track for each of 18 cell types.

For older assemblies you can load, they can be browsed below:

<https://segway.hoffmanlab.org/>

# bioconda / packages / segway 2.0.2



0

a tool for easy pattern discovery and identification in functional genomics data.

Conda

Files

Labels

Badges

 License: [GPL2](#) Home: <http://segway.hoffmanlab.org/>

1706 total downloads

Last upload: 3 months and 29 days ago

## Installers

`conda install`

linux-64 v2.0.2

osx-64 v2.0.2

To install this package with conda run:

```
conda install -c bioconda segway
```

Atlassian, Inc. (US) https://bitbucket.org/hoffmanlab/segway/src/ Search

Hoffman / segway / segway / Source

## Source

default ▾ [segway /](#)

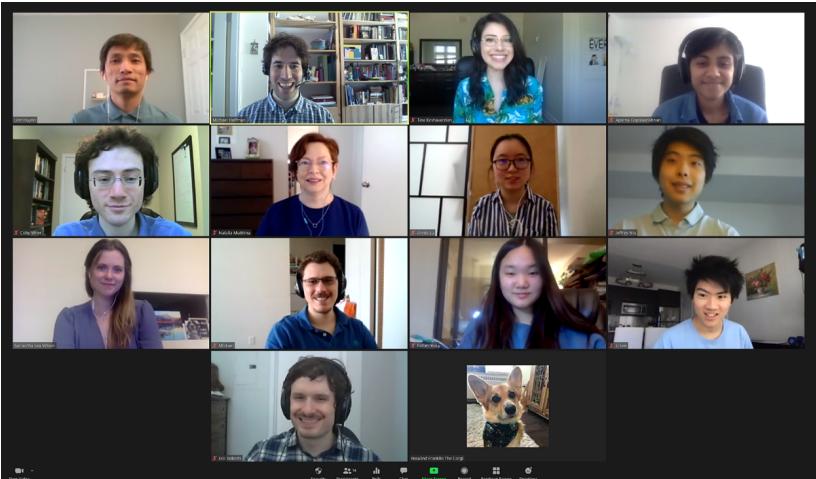
- doc
- docker
- segway
- test

<a href="#">.hgignore</a>	128 B	2014-10-18	.hgignore: add .vcallrc
<a href="#">.hgtags</a>	3.4 KB	2018-10-04	Added tag 2.0.3 for changeset 50f116fc1fd
<a href="#">CONTRIBUTING.rst</a>	3.0 KB	2016-04-28	Fix PEP8 formatting and update contributing guidelines
<a href="#">LICENSE</a>	18.1 KB	2015-10-19	Add GPLv2 License copy
<a href="#">NEWS</a>	30.9 KB	2018-10-05	Update version number for next release
<a href="#">README.rst</a>	2.3 KB	2018-10-22	Rename README to README.rst to hint Bitbucket parsing
<a href="#">bitbucket-pipelines.yml</a>	975 B	2018-09-28	Update test pipeline to include set python version if provided
<a href="#">setup.cfg</a>	26 B	2015-01-23	Remove TODO. Remove binary distribution from setup configuration.
<a href="#">setup.py</a>	6.0 KB	2018-09-11	Update colorbrewer package dependency for Python 2 and 3 compatibility

## Segway

# Acknowledgments

## The Hoffman Lab



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Leo Li

Natalia Mukhina

Linh Huynh

Coby Viner

Jeffrey Niu

**Aparna Gopalakrishnan**

Esther Yu

Jeff Bilmes

William Noble

Max Libbrecht

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Princess Margaret Cancer Foundation

Canadian Institutes of Health Research

Canadian Cancer Society

Natural Sciences and Engineering Research Council

Ontario Institute for Cancer Research

Ontario Ministry of Research, Innovation and Science

Medicine by Design

McLaughlin Centre

**Slides:** <https://tinyurl.com/segwaygmtk>



Postdoctoral, MSc, PhD positions  
available in my research lab at the

**Princess Margaret Cancer Centre**

**Dept of Medical Biophysics  
Dept of Computer Science  
University of Toronto**

Please approach me for details.

Michael Hoffman  
<https://hoffmanlab.org/>

michael.hoffman@utoronto.ca  
@michaelhoffman



# Software availability

- Segway
  - data tracks → segmentation
  - Hoffman MM *et al.* 2012. *Nat Methods* 9:473.
  - <https://segway.hoffmanlab.org/>
- Segtools
  - segmentation → plots and summary statistics
  - Buske OJ *et al.* 2011. *BMC Bioinformatics* 12:415
  - <https://segtools.hoffmanlab.org/>
- Genomedata
  - efficient access to numeric data anchored to genome
  - Hoffman MM *et al.* 2010. *Bioinformatics* 26:1458.
  - <https://genomedata.hoffmanlab.org/>
- Umap and Bismap
  - Identify unmappable regions and corresponding missing data
  - Karimzadeh *et al.* 2018. *Nucleic Acids Res*, in press. <https://doi.org/10.1093/nar/gky677>
  - <https://bismap.hoffmanlab.org/>

# GMTK Core Features

1. Textual Graph Language
2. Switching Parent Functionality
3. Forwards and Backwards time links
4. Multi-rate models with extended templates.
5. Linear Dependencies on observations
6. Non-linear (Deep Many-Layered) Unary Potentials
7. Arbitrary low-level parameter sharing (EM/GEM training)
8. Gaussian Vanishing/Splitting algorithm.
9. Decision-Tree and C++ Based implementations of dependencies (deterministic, sparse, formula leaf nodes)
10. Full inference, single pass decoding, online inference
11. Linear and Island Algorithm ( $O(\log T)$ ) Exact Inference

# Graphical Models Toolkit (GMTK)

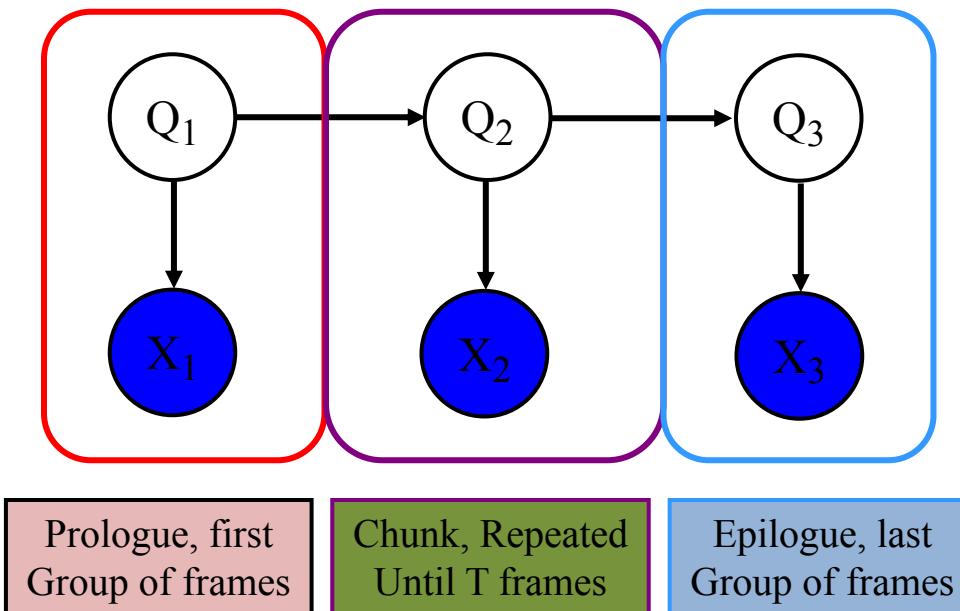
1. Textual Graph Language
2. Switching Parent Functionality
3. Forwards and Backwards time directed edges
4. Edges across multiple frames and chunks
5. Multi-frame and multi-rate models with extended DBN templates.
6. Disconnected Networks
7. Static Networks (in addition to dynamic)
8. Native ARPA Language Models
9. Full HTK lattice support with additional support for soft bounded node range determination.
10. Native Factored Language Models (compatible with all SRILM-FLM generalized backoff options)
11. Linear Dependencies on observations
12. Feature-file processing (up/down sampling, merging, subset selection, etc.)
13. Arbitrary low-level parameter sharing (EM/GEM training)
14. Gaussian Vanishing/Splitting algorithm.
15. Decision-Tree-Based implementations of dependencies
16. Rich and arbitrary integer formula DT leaf nodes.
17. Full inference, single pass decoding
18. Sampling Methods
19. Linear and Island Algorithm ( $O(\log T)$ ) Exact Inference
20. Native Virtual Evidence (Hybrid {SVM,ANN}/DBN)
21. Virtual Evidence Separators (speeds VE)
22. Switching Weights (penalties, scales, and shifts)
23. Separate GMTK Triangulation Engine (\*many\* heuristics supported)
24. Anytime triangulation search
25. Graph visualization tool (gmtkViz)
26. Tieing program (gmtkTie), Simon King
27. Fancy debugging/trace output to help debugging your graph.
28. Much much faster! (up to 800 times!!)

# ... under the hood:

- Algorithmic features:
  1. triangulation of DBN partitions rather than entire unrolled graph
  2. Optimal boundary/separator determination
  3. Max-flow (and submodular) based boundary determination
  4. Big cliques can be fast cliques (when determinism abounds). Non-elimination based triangulation.
  5. Separator (rather than clique) driven inference to quickly and dynamically remove zeros. Yields exact and approximate inference (via dynamic pruning)
  6. Virtual Evidence Separators (utilize constraints)
- System level features:
  1. Topological and ascending cardinality ordered variable iteration within cliques
  2. Almost optimal packed representations
  3. DBN clique-specific value sharing
  4. Anytime triangulation search
  5. No C++ STL for inner loops (custom data structures, PHiPAC-style optimizations, and memory management)

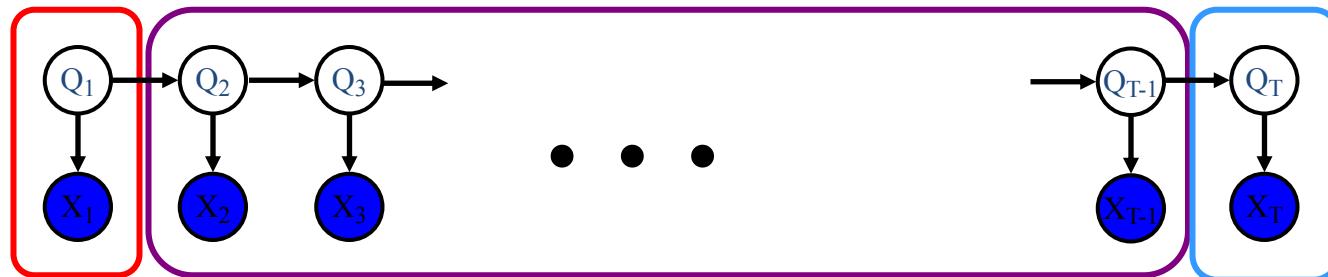
# GMTK Structure file for HMM

- Structure file defines a prologue P, chunk C, and epilog E. E.g., for the basic HMM:



# GMTK Unrolled structure

- Chunk is unrolled  $T$ -size(prologue)-size(epilog) times (if 1 frame in chunk)



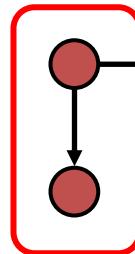
Prologue, first  
group of frames

Chunk, Repeated  
until  $T$  frames is obtained.

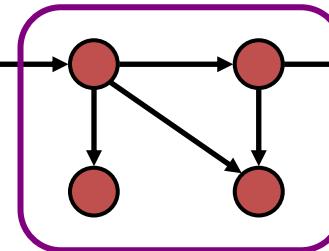
Epilog, last  
group of frames

# Multiframe Repeating Chunks

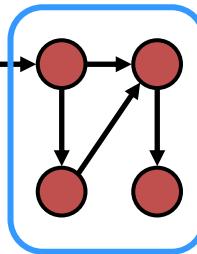
Prologue P



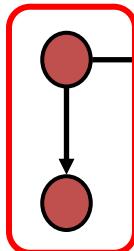
Repeating Chunk C



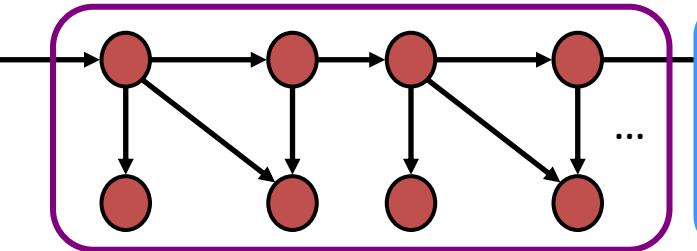
Epilogue E



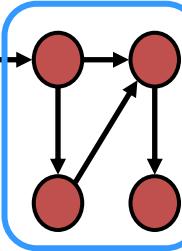
Prologue



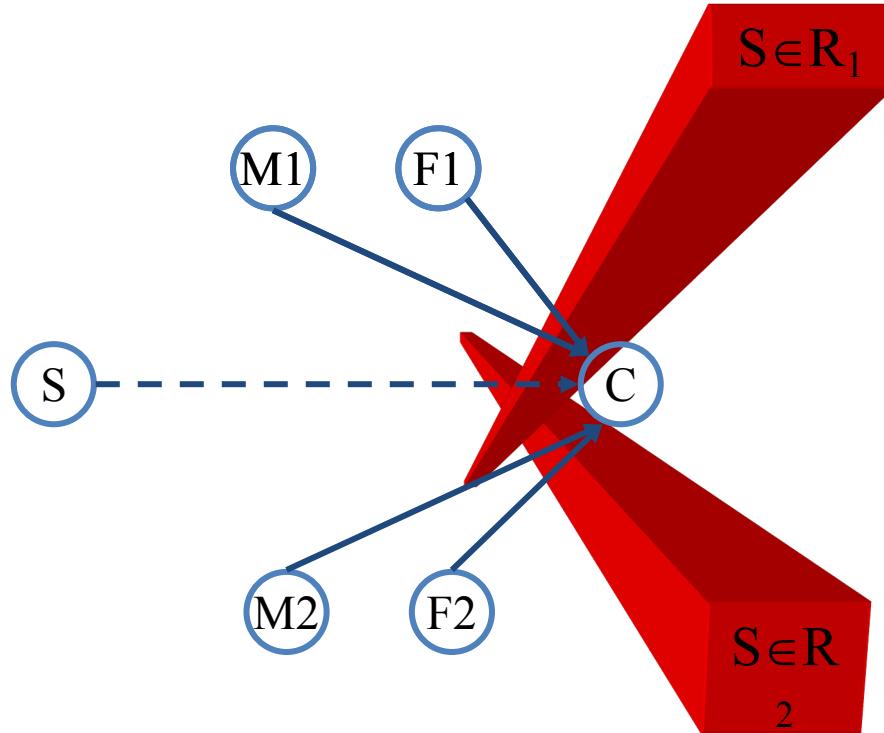
Chunk Unrolled 1 time



Epilogue



# Switching Parents



$$P(C | M_1, F_1, M_2, F_2) = \sum_i P(C | M_i, F_i, S \in R_i) P(S \in R_i)$$

# Switching Parents

- Switching parents can also be used to switch CPT implementations
- Here, depending on the value of wordTransistion(-1), word(0) will use either a “copy” or will use a “wordBigram” as the CPT.

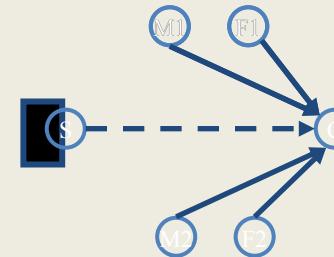
```
variable : word {  
    type: discrete hidden cardinality VOCAB_SIZE;  
    switchingparents: wordTransition(-1)  
        using mapping("directMappingWithOneParent");  
    conditionalparents:  
        word(-1) using DeterministicCPT("copyCPT")  
        | word(-1) using DenseCPT("wordBigram");  
}
```

# GMTK Switching Structure

```
variable : S {
    type : discrete hidden cardinality 100;
    switchingparents : nil;
    conditionalparents : nil using DenseCPT("pi");
}

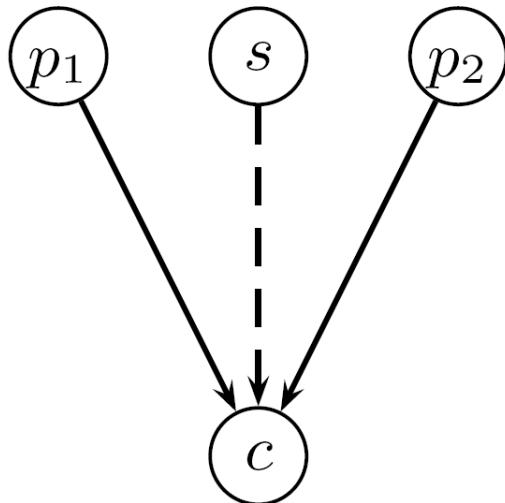
variable : M1 { ... }
variable : F1 { ... }
variable : M2 { ... }
variable : F2 { ... }

variable : C {
    type : discrete hidden cardinality 30;
    switchingparents : S(0) using mapping("S-mapping");
    conditionalparents :
        M1(0),F1(0) using DenseCPT("M1F1")
        | M2(0),F2(0) using DenseCPT("M2F2");
}
```



# Switching Parents

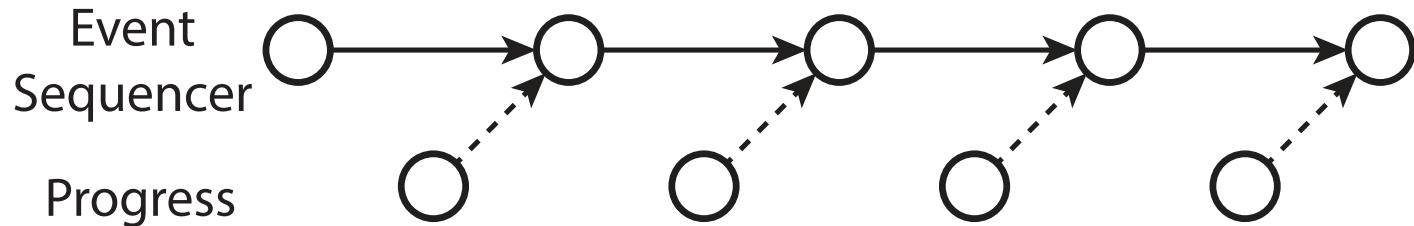
```
variable : C {  
    type: discrete hidden cardinality STATES;  
    switchingparents:  
        S(0)using  
mapping("directMappingWithOneParent");  
    conditionalparents:  
        p1(0) using DenseCPT("probGivenP1") |  
        p2(0) using DenseCPT("probGivenP2");  
}
```



Switching parents used here  
to switch between parents but  
this is not all that they can do ...

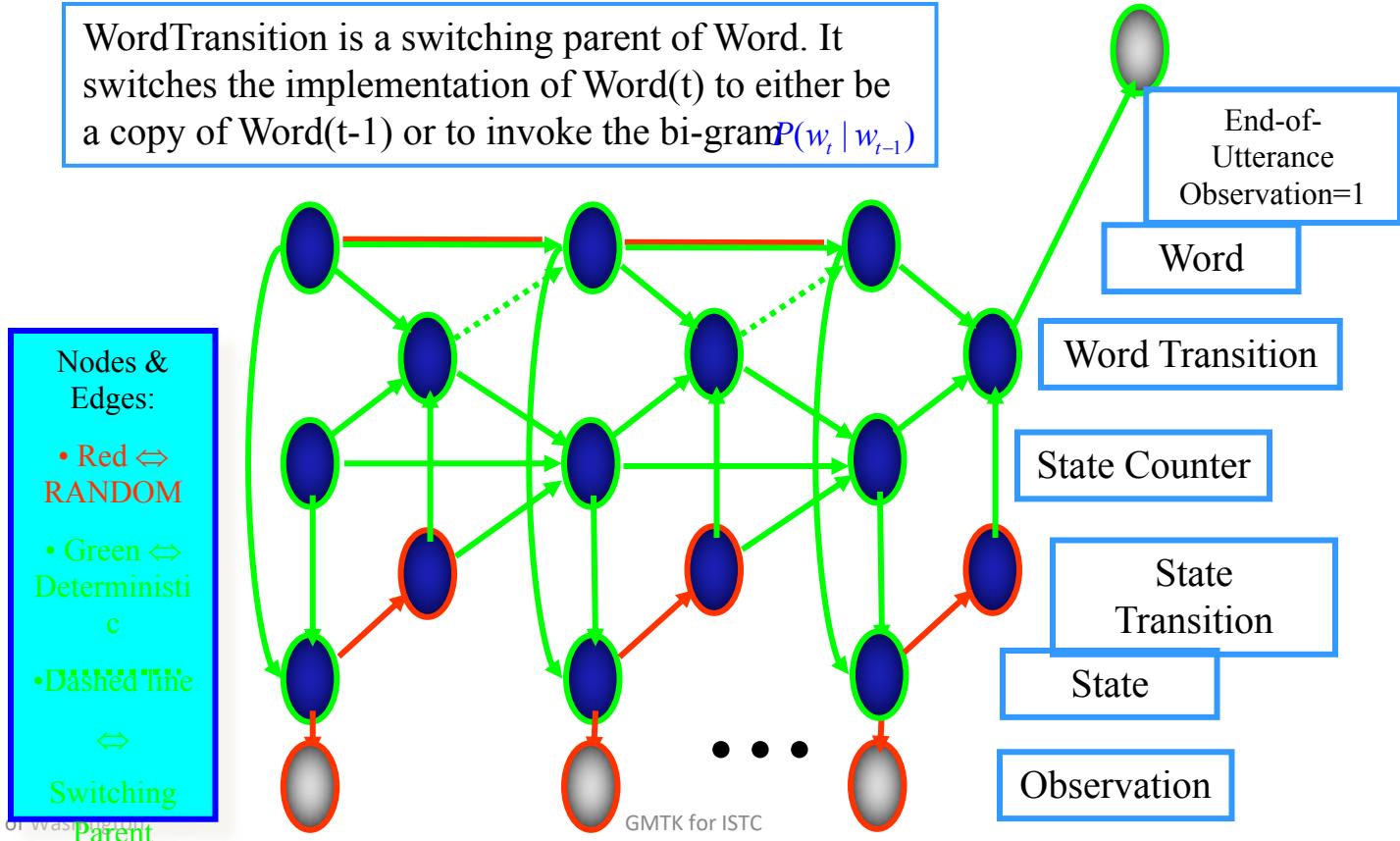
# Stochastic Event Sequencer w. Switching Parents

- Event sequencer, sequences through an order of events.
- Binary progress random variable controls progress.

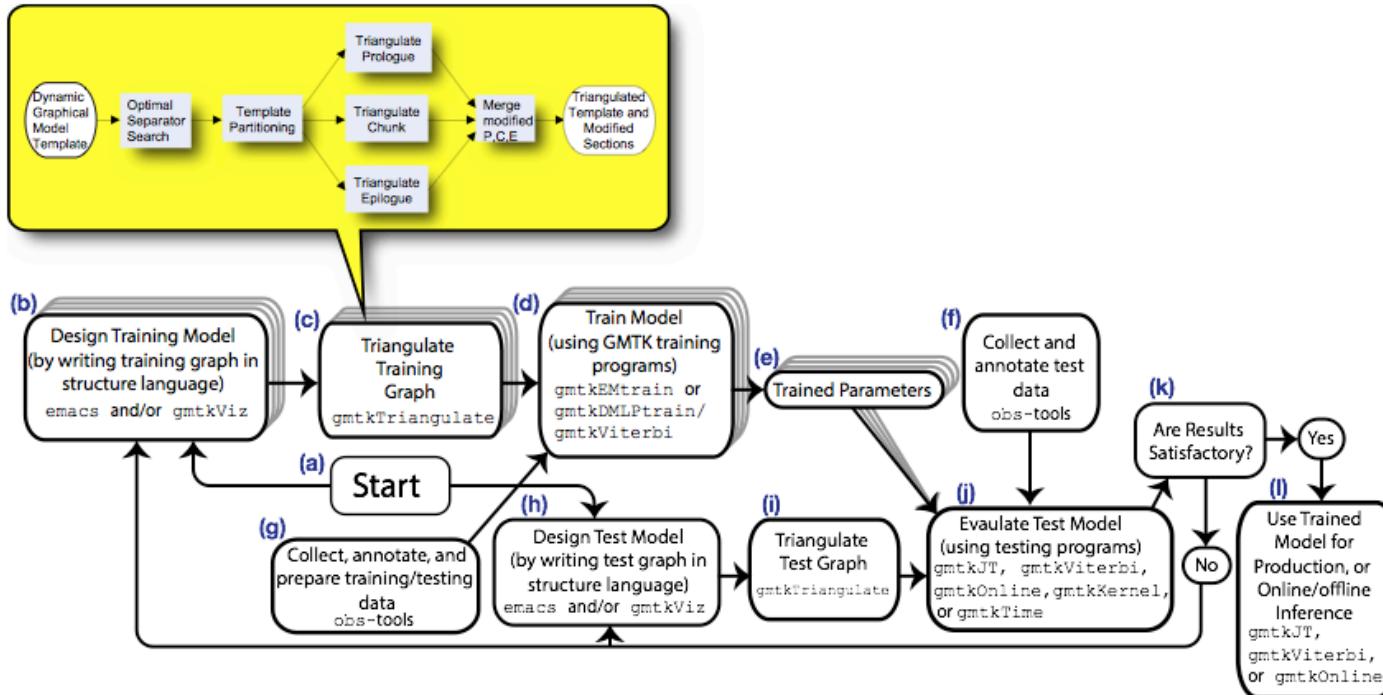


# Explicit bi-gram Decoder

WordTransition is a switching parent of Word. It switches the implementation of Word(t) to either be a copy of Word(t-1) or to invoke the bi-gram  $P(w_t | w_{t-1})$



# GMTK Model Design/Use Workflow



# Parameter Definitions - MasterFile

- Parameters are stored either via a MasterFile or via a “trainable file”
- For each parameter either
  - It is stored directly in the MasterFile
  - MasterFile gives a pointer to the location of the file where the parameters are stored
  - The master file can include all the parameters that could be trained.
- Input/Output Trainable parameters
  - The trainable file is a short-cut to save/restore parameters that might change during training.

# Parameter Definitions - CPTs

- There are many types of CPTs in GMTK, which define the values used for  $p(\text{child} \mid \text{parents})$ .
- The types of CPTs used in the TIMIT tutorial include:
  - Dense – all values of  $p(a \mid b)$  are included, leading to  $O(r^K)$  sized table
  - Deterministic – ‘a’ is deterministically related to its parents, so what is specified is that functional relationship
- There are many others as well, including
  - Sparse – ‘a’ is such that only a sub-set of ‘a’ s possible values are specified, i.e., the CPT is sparse.
  - Language/factored language model, virtual evidence
  - Deep (many-layered neural network) CPTs

# Deterministic CPT

- Value of child given its parents is a deterministic function
- $A=f(B)$ ,  $p(A|B)=1$  or  $p(A|B)=0$
- Decision trees (with functional leaves) are used for specifying the deterministic function  $f(B)$  (these will be mentioned soon)

```
wordPosition
3          % number of parents
2 2 13 13      % cardinalities of parents, self
wordPosition_DT % decision tree
```

# GMTK's Decision Trees

- Decision trees - for specifying arbitrary deterministic functions: integer to integer mappings
- Leaf-node formulas (including conditionals and functions).
- Can also be expressed directly in C++ for efficiency.
- Example:

```
wordPositionDT  % Name
3                  % Number of Parents
1 2 0 default
    -1 { p2+1 }
0 2 1 default
    -1 0
    -1 7
```

```
phoneCounterDT  % Name
2                  % Number of Parents
-1 { min(p0+1, 7*(p1/4)-3, mc) }
```

- Can also be expressed directly in C++ for efficiency.

# Gaussian and Conditional Gaussian Parameters

- Mixture of components
    - Gives list of components
  - Component
    - Specifies mean and variance if Gaussian
  - Mean, Variance, DPMF (mixture weights)
  - Also, dlinks for sparse autoregressive multivariate Gaussians
- Defined in this way so that anything can be shared

# EM Training

- The EM (expectation/maximization) algorithm is used for maximum-likelihood parameter training.
- Useful when there is missing values.
- `gmtkEMtrain`
  - The program that reads in a graph and initial set of parameters, and iterates via the EM (expectation-maximization) algorithm.
- Discriminative training also possible (but much slower, not part of tutorial currently)

# EM Training

- Gaussian Splitting and Vanishing
  - Gaussians are initialized as single component, zero mean, unit variance
  - Single components trained
  - Single components are split into two components, then trained
  - Repeat
- Optionally split only components with high mixture weights
- Optionally vanish components with low mixture weights

# Viterbi Decoding

- gmtkViterbi
  - Also called MPE (most probable explanation)
  - This is what is used for decoding, you run this to get, say, a list of activities/phones/etc. based on what the input observations
  - Implements:

$$(h_{1:T}^*, y_{1:T}^*) \in \underset{h_{1:T}, y_{1:T}}{\operatorname{argmax}} \Pr(\bar{x}_{1:T}, h_{1:T}, y_{1:T})$$

# ChromHMM vs. Segway

	ChromHMM	Segway
Modeling framework	Hidden Markov model	Dynamic Bayesian network
Genomic resolution	200 bp	1 bp
Data resolution	Boolean	Real value
Handling missing data	Interpolation	Marginalization
Emission modeling	Bernoulli distribution	Gaussian distribution
Length modeling	Geometric distribution	Geometric distribution plus hard and soft constraints